Multibody Analysis Comparison Between Strain-Based Beam Formulation and Absolute Nodal Coordinate Formulation

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Abstract

The wings of recent regional jet tend to have high aspect ratio to reduce induced drag [1]. Because of the high aspect ratio configuration with lightweight, the wings undergo large deformations induced by aerodynamic forces. In addition, such high aspect ratio wings experience very high wing root bending moment when they encounter gust. To alleviate the gust response, the researchers in University of Bristol and the engineers in Airbus [2] have developed a folding wing concept. When designing the high aspect ratio wings with the folding wing mechanism, a multibody simulation considering geometric nonlinearity is necessary. Owing to the slenderness of the high aspect ratio wing, geometrically nonlinear beam formulations are suitable for the simulation.

Palacios et al. [1] reviewed the three representative geometrically nonlinear beam formulations often used for the high aspect ratio wing simulation. The first one is the displacement/rotation-based geometrically nonlinear beam formulation developed by Simo and Vu-Quoc [3]. In this formulation, the nonlinearity appears on both the inertial and elastic forces. This formulation has been widely used for multibody simulation. The second one is the fully intrinsic beam formulation developed by Hodges [4]. This formulation uses internal forces and translational and rotational velocities as variables. The remarkable advantage of this formation is that only quadratic nonlinearity appears in the equation. This formulation was extended to multibody analysis by Wang and Otsuka [5]. The third one is the strain-based beam formulation developed by Cesnik and Brown [6]. This formulation uses the curvatures and strain as variables. The nonlinearity appears only on the inertial force, which means the stiffness matrix is constant. On the other hand, absolute nodal coordinate formulation (ANCF) [7] developed for multibody simulation has a constant mass matrix. Because ANCF uses position and gradient vectors as generalized coordinates, various multibody constraints can be written in simple linear or quadratic nonlinear forms. Otsuka et al. [8, 9] have made ANCF the new option for the aircraft simulation. Recently, Otsuka et al. [10] have found that ANCF can be transformed to the strain-based beam formulation via velocity transformation. In this study, we examine the accuracy of the transformed strain-based beam formulation by the comparison with ANCF in multibody simulation.

The equation of motion in ANCF is written as

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\frac{\partial \boldsymbol{\Phi}}{\partial \mathbf{q}}\right)^{\mathrm{T}} \boldsymbol{\lambda} = \mathbf{F} - \mathbf{F}_{\mathrm{elastic}}. \quad (1)$$

$\mathbf{M}$ is a constant mass matrix. $\mathbf{F}$ is an external force vector. $\mathbf{F}_{\mathrm{elastic}}$ is a nonlinear elastic force vector. $\boldsymbol{\Phi}$ is a constraint equation vector. $\boldsymbol{\lambda}$ is a Lagrange multiplier vector. The generalized coordinate is defined as

$$\mathbf{q} \equiv \left[\left({}^1\mathbf{r}^1\right)^{\mathrm{T}} \quad \left({}^1\mathbf{r}_x{}^1\right)^{\mathrm{T}} \quad \left({}^1\mathbf{r}_y{}^1\right)^{\mathrm{T}} \quad \left({}^1\mathbf{r}_z{}^1\right)^{\mathrm{T}} \quad \cdots \quad \left({}^j\mathbf{r}^i\right)^{\mathrm{T}} \quad \left({}^j\mathbf{r}_x{}^i\right)^{\mathrm{T}} \quad \left({}^j\mathbf{r}_y{}^i\right)^{\mathrm{T}} \quad \left({}^j\mathbf{r}_z{}^i\right)^{\mathrm{T}} \right]. \quad (2)$$

The superscripts i and j represent the ith node of the jth body, respectively. The strain-based beam formulation can be obtained from ANCF by the velocity transformation matrix $\mathbf{B}$. The equation of motion is written as

$$\mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{B}\ddot{\mathbf{p}} + \mathbf{B}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{B}}\dot{\mathbf{p}} + \mathbf{K}\mathbf{p} + \left(\frac{\partial \boldsymbol{\Phi}}{\partial \mathbf{p}}\right)^{\mathrm{T}} \boldsymbol{\lambda} = \mathbf{B}^{\mathrm{T}}\mathbf{F}. \quad (3)$$

$\mathbf{K}$ is a constant stiffness matrix. The generalized coordinate is defined as

$$
\mathbf{p} \equiv \Big[\left({}^{1}\mathbf{r}^{1}\right)^{\mathrm{T}} \quad \left({}^{1}\mathbf{r}_{x}^{1}\right)^{\mathrm{T}} \quad \left({}^{1}\mathbf{r}_{y}^{1}\right)^{\mathrm{T}} \quad \left({}^{1}\mathbf{r}_{z}^{1}\right)^{\mathrm{T}} \quad {}^{1}\varepsilon^{1} \quad {}^{1}\kappa_{x}^{1} \quad {}^{1}\kappa_{y}^{1} \quad {}^{1}\kappa_{z}^{1} \quad \cdots
$$
$$
\left({}^{j}\mathbf{r}^{1}\right)^{\mathrm{T}} \quad \left({}^{j}\mathbf{r}_{x}^{1}\right)^{\mathrm{T}} \quad \left({}^{j}\mathbf{r}_{y}^{1}\right)^{\mathrm{T}} \quad \left({}^{j}\mathbf{r}_{z}^{1}\right)^{\mathrm{T}} \quad \cdots \quad {}^{j}\varepsilon^{i} \quad {}^{j}\kappa_{x}^{i} \quad {}^{j}\kappa_{y}^{i} \quad {}^{j}\kappa_{z}^{i} \Big]^{\mathrm{T}}. \tag{4}
$$

The rigid body motion of the strain-based beam is described by the position and direction vectors of the root node, namely $\mathbf{r}^1$, $\mathbf{r}_x^1$, $\mathbf{r}_y^1$, and $\mathbf{r}_z^1$. The elastic deformation of the strain-based beam is described by the extensional strain ε, torsional curvature κ_x, bending curvatures κ_y and κ_z.

By using the strain-based beam formulation and ANCF, a double pendulum motion in Fig. 1 was simulated. The double pendulum has a rectangular cross section with a width of 0.02 m and a height of 0.02 m. Young's modulus is 0.02 GPa. Material density is 7200 kg/m^3. Initially, the double pendulum was placed on the X-axis. The gravitational force causes free falling motion. Figure 2 shows the time history of the Z coordinate at the pendulum free end. The strain-based beam formulation and ANCF are in good agreement.

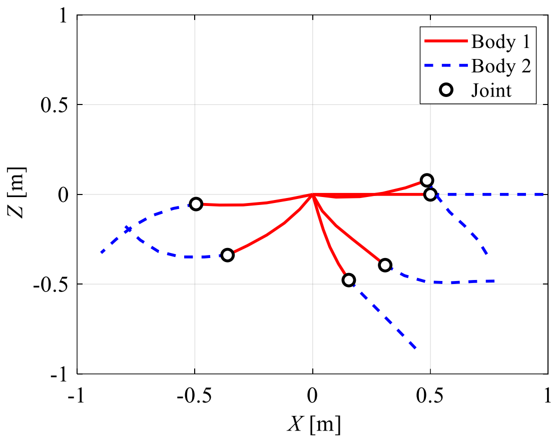

Figure 1: Double pendulum motion.

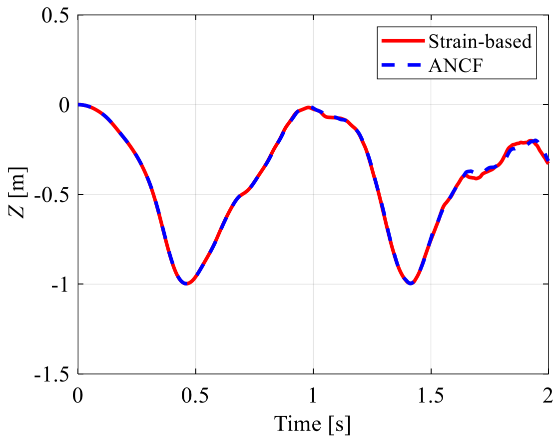

Figure 2: Z coordinate at free end.

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