Balanced Exploration and Exploitation Properties of Multi-Objective Sooty Tern Optimizer

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***Abstract*—The Sooty Tern Optimization Algorithm (STOA) [1], a newly designed metaheuristic drawing inspiration from the migratory and predatory behaviors of the sea bird Sooty Tern, presents several notable advantages, including minimal parameterization. However, it is confined to addressing single-objective problems exclusively. In this work, we have enhanced the properties of exploration and exploitation to effectively penetrate the area of search. Subsequently, we extended the STOA to a multi-objective version called MOSTOA (Multi-Objective Sooty Tern Optimization Algorithm). This enhanced algorithm is designed to address multiple objectives in diverse problem domains. The MOSTOA utilizes an archive repository to store and retrieve the optimal solutions generated throughout the optimization cycle. From this population archive, leaders are chosen to guide the solutions of the main population towards promising search locations. Furthermore, the utilization of the grid mechanism and dynamic archiving approach serves the purpose of achieving a harmonious equilibrium between convergence and variety inside the final Pareto set. These strategies ensure that the obtained solutions exhibit both high quality and spread across the objective space. The proposed MOSTOA is validated on various well-known benchmarks functions. In addition, its performance is assessed in comparison to well-established cutting-edge algorithms. Our method produces very competitive results and, in most circumstances, exhibits improved convergence behavior with a good variety of solutions, as demonstrated by the experimental findings.**

Keywords— Multi-Objective optimization, Swarm Intelligence, Dominance relation, Metaheuristic, Pareto Front

I. INTRODUCTION

In recent years, there have been many multi-objective optimizers proposed with the purpose of addressing multi-objective optimization problems(MOPs), such as: MODA [1], MOMFO [2], MOALO [3], MOGWO [4], MOCSO[5], MOHHO [6] ...etc. As a consequence of the contradictory characteristics of objectives, there is generally no one optimum solution in multi-objective optimization, but instead a set of proposal known as Pareto optimal (Ps) [7]. Since no other solutions exist in the area search that are superior for all of the objectives covered, these solutions are considered optimum. As expected, significant challenges were posed that need specific strategies to handle them, including the harmony between the exploration and the exploitation properties, since these are the essential criteria in multi-objective optimization. In this paper, we present our suggested algorithm based on an equilibrium of exploration and exploitation properties. The remainder of this study is arranged in the following manner: Section 2 offers the core concepts of MOPs and provides a brief background on STOA. Section 3 presents our innovative Multi-Objective Sooty Tern Optimization Algorithm (MOSTOA). Section 4 contains the results of the experiment and discussions, while Section 5 summarizes our findings and offers recommendations for future research.

II. BACKGROUND

A. Multi-Objective Optimization

The typical structure of a MOP can be characterized in the following manner:

$$\begin{aligned} &Maximize: f_m(x), \quad m = 1,2,....M \\ &Subject\,to: g_j(x) = 0, \;\; j = 1,2,...J \\ &\qquad\qquad h_k(x) = 0, \;\; k = 1,2,...K \\ &\qquad\qquad L_i \le x_i \le U_i, i = 1,2,..n \end{aligned} \tag{1}$$

In this context, x represents a solution comprising n choice individuals (x_1, x_2, ..., x_n) that must adhere to J and K constraints. The function M quantifies the number of objectives, while L_i and U_i denote the minimum and maximum bounds for each decision individual.

B. Pareto dominance

Dominance relation. A solution $x^{(i)}$ is considered to dominate another solution $x^{(j)}$, represented as $x^{(i)} \prec x^{(j)}$ when two specific conditions hold true [8] :

- $\forall m \in \{1,..., M\}: f_m\left(x^{(i)}\right) \le f_m\left(x^{(j)}\right)$
- $\exists m \in \{1,...., M\} \; : f_m\left(x^{(i)}\right) < f_m\left(x^{(j)}\right)$ (2)

Non-dominated collection. Within a given solution collection, the non-dominated solutions can be defined as those within the subset $A' \subseteq A$ that are not outperformed by any other member within the same collection A [9].

Pareto solution collection. The collection of non-dominated solutions within the entire area search S is referred to as the Pareto optimal set [10].

C. Sooty Tern Optimization Algorithm (STOA)

Sooty terns, as omnivorous seabirds, sustain themselves through a diverse diet comprising earthworms, insects, and fish. These avian species tend to congregate in colonies and employ swarm intelligence to efficiently locate and capture their prey.

Within the realm of the sooty tern's predation process, migration and attack emerge as two pivotal behaviors, offering essential heuristic foundations for constructing the STOA [11]. The subsequent section delves into the mathematical representations of these migration and attack behaviors

Exploration Phase. Sooty terns embark on their migratory journeys primarily to mitigate the risk of mid-air collisions, a phenomenon that can be succinctly framed as:

$$L_{st} = K_A \times R_{st}(t) \tag{3}$$

$$K_A = L_f - (t \times (L_f / MaxT)) \tag{4}$$

In this context, L_{st} represents the desired position of a search individual to avoid collisions with others. $R_{st}(t)$ denotes the current position of the search individual at iteration t, where K_A signifies movement within the search space. The variable L_f serves as a control parameter responsible for progressively reducing the search amplitude (K_A) in a linear manner from its initial value L_f to zero.

Following avoiding collisions, the search individuals efficiently go towards the vicinity of the optimal individual by employing the subsequent mathematical expressions:

$$D_{st} = L_B \times (R_{bst}(t) - R_{st}(t)) \tag{5}$$

$$L_B = 0,5 \times Rd \tag{6}$$

D_{st} quantifies the positional disparity between the search individual R_{st} and the best individual R_{bst} . Meanwhile, L_B plays a pivotal role in improving exploration and is determined by a uniformly distributed random variable Rd , which ranges between 0 and 1.

In the end, every individual has the opportunity to adjust its position based on the best search individual.

$$E_{st} = L_{st} + D_{st} \tag{7}$$

E_{st} signifies the disparity between the individual and the fittest individual.

Exploitation Phase. During their hunt for prey, sooty terns display a distinctive spiral flight pattern, which can be characterized as follows:

$$s_1' = sin(j) \times Rad \tag{8}$$

$$s_2' = \cos(j) \times Rad \tag{9}$$

$$s_3' = j \times Rad \tag{10}$$

$$Rad = a \times e^{hb} \tag{11}$$

In this context, Rad denotes the distance between successive turns in a spiral, 'i' is a number within $[0 \le h \le 2\pi]$, and 'a' and 'b' are constants that shape the spiral, both set to 1 for our analysis. Subsequently, we use the following equation to update the search individual's position:

$$R_{st}(t+1) = (E_{st} \times (s_1' + s_2' + s_3')) \times R_{bst}(t) \tag{12}$$

In the context of this study, the term '$R_{st}(t + 1)$' denotes the process responsible for updating the positions of other search individuals while ensuring the preservation of the optimal solution.

III. The Extended Sooty Tern Optimization Algorithm for Addressing the MOP

This part outlines the proposed Multi-Objective Sooty Tern Optimization Algorithm (MOSTOA), designed for tackling MOPs. In MOP, the effectiveness of an algorithm hinges significantly on its elitism strategies, which involve preserving the historical record of Pareto solutions. MOSTOA employs an external archive to store non-dominated solutions acquired during optimization. This archive houses the Pareto front, necessitating careful updates throughout the optimization process to balance both convergence and diversity. Moreover, the selection of the global best solution, denoted as R_{bst} , plays a pivotal role in MOSTOA. It directly influences the movements of sooty tern within the search space, thereby impacting the exploitation of promising regions.

A. *The archival update technique*

The archive update mechanism serves to distinguish potential solutions within the main population, separating accepted and rejected ones. To achieve this, MOSTOA combines the current population and archive into a temporary repository. From this repository, it identifies and stores the set of non-dominated solutions into the archive. However, this may lead to a new Pareto set that surpasses the maximum archive size (Tmax). In such cases, an adaptive grid cells strategy, similar to the one outlined in [12], is employed as a density estimator. Its purpose is to remove the most crowded solutions (Tmax - |archive|) from the archive, with the aim of retaining well-distributed solutions in the search space. For a visual representation of this archiving strategy, refer to Fig. 1.

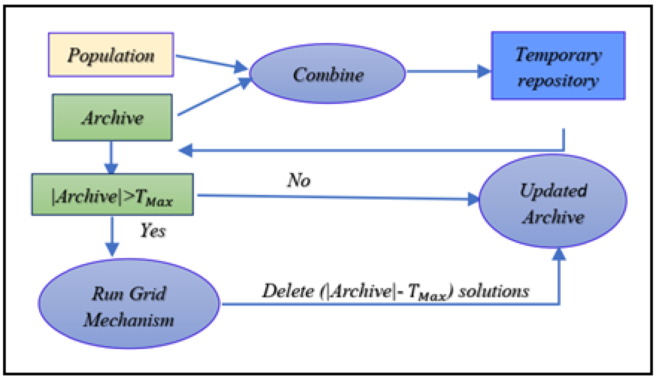

Fig. 1. Flowchart of the archive update mechanism

The grid strategy serves the purpose of partitioning the external archive into a specific number of hypercubes, with each hypercube containing a defined number of solutions. This mechanism functions as a density estimator, with the quantity of solutions within a particular hypercube representing the density of that hypercube. Additionally, the density of a particular solution corresponds to the density of the hypercube it belongs to. This density measurement plays a critical role in identifying both the sparsely populated and densely populated hypercubes. New non-dominated solutions are inserted into the less crowded hypercubes, while certain solutions are removed from the densely populated ones when the archive reaches its capacity. Consequently, this approach helps maintain a well-distributed set of solutions within the archive.

B. *Global best solution (R_{bst}) selection approach*

In our proposed multi-objective algorithm, a key strategy revolves around the selection of the global best solution, denoted as R_{bst}. This particular individual plays a pivotal role in guiding its fellow solutions towards promising areas within the search space, with an emphasis on achieving both convergence and diversity.

To identify R_{bst} in the MOSTOA, we adopt a method that involves selecting it from the less densely populated hypercubes in the objective space. This selection process is based on a probability calculation using the following formula:

$$P_i = v / T_i \tag{13}$$

Here, 'v' represents a constant value greater than 1, while 'T_i' signifies the number of solutions contained within the i^{th} hypercube. Essentially, this probability calculation favors hypercubes with fewer solutions, effectively prioritizing less crowded hypercubes for selection.

Furthermore, once an appropriate hypercube is chosen using this method, a roulette-wheel selection mechanism is employed to randomly pick an individual as the R_{bst}. This multi-step approach ensures that R_{bst} is selected with consideration for both convergence and diversity, enhancing the algorithm's overall performance in multi-objective optimization tasks.

C. *The Pseudocode of MOSTOA algorithm*

```
Algorithm:    Multi-Objective-STOA

Initialize the population of Sooty Tern: X_i; i=1...N;
Evaluate each solution in X_i ;
Set T_max as the size of the archive population;
Initialize the archive (Arch) with the non-dominated
solution from X_i;
While t< MaxT do
    For i=1 to N do
        Select a global best solution R_bst;
        Update the positions of individuals using Eq.12
    End
    Update the parameters K_A, and L_B
    Evaluate the new population
    Update the current archive (Arch)
    If Arch is full then
        Run the grid strategy;
        Remove the last (T_max-|archive|) solutions;
    End
End
Return the final archive Arch
```

IV. RESULTS AND DISCUSSION

In assessing the effectiveness of the novel Multi-Objective Sooty tern optimization algorithm (MOSTOA), a set of benchmark functions from the ZDT series [13] is employed.. To gauge the competitiveness of our approach, a comprehensive qualitative and quantitative evaluation is conducted in comparison to three established algorithms in the field, namely, MOEA/D [14], MOGWO [4], and MSSA [15] . For the benchmarking process, a uniform configuration is maintained across all algorithms, encompassing a population size of 100, an archive size of 100, and a fixed number of generations set at 1000.

To enable a rigorous quantitative comparison, two widely recognized performance metrics, namely Inverted Generational Distance (IGD) [16] and Hypervolume (HV) [17], are harnessed. IGD serves as a measure of convergence, offering insights into the algorithm's ability to approach the true Pareto front, while HV quantifies the diversity within the obtained Pareto front. These metrics collectively provide a comprehensive assessment of MOSTOA's performance against its contemporaries.

A. *The finding of ZDT test functions*

The statistical outcomes, encompassing the best, worst, mean, median, and standard deviation (std) values of the IGD and HV metrics derived from the algorithms, are presented in Tables I and II, with the superior outcomes highlighted in bold. Furthermore, in Figures 2 and 3, we visually depict the most optimal Pareto fronts obtained by each algorithm.

TABLE I. COMPARATIVE RESULTS OF IGD METRIC

		MOSTOA	MOGWO	MSSA	MOEAD
ZDT1	best	**8,74E-05**	6,10E-03	7,20E-03	3,14E-03
	worst	**8,00E-05**	5,89E-04	3,15E-03	5,41E-04
	mean	**8,32E-05**	1,50E-03	4,73E-03	1,15E-03
	median	**8,29E-05**	1,29E-03	4,46E-03	7,71E-04
	std	**1,92E-06**	9,63E-04	1,06E-03	7,65E-04
ZDT2	best	**2,52E-04**	2,31E-02	1,16E-02	6,24E-02
	worst	**8,90E-05**	7,93E-04	3,65E-03	3,99E-04
	mean	**1,07E-04**	6,45E-03	5,30E-03	2,20E-02
	median	**9,79E-05**	1,59E-03	5,10E-03	1,86E-02
	std	**3,03E-05**	9,16E-03	1,43E-03	1,54E-02
ZDT3	best	**1,33E-04**	3,26E-03	1,16E-02	2,39E-02
	worst	**9,21E-05**	3,06E-04	4,50E-03	6,03E-03
	mean	**1,07E-04**	9,66E-04	7,18E-03	1,04E-02
	median	**1,05E-04**	8,02E-04	6,93E-03	1,03E-02
	std	**8,91E-06**	5,93E-04	1,78E-03	3,07E-03
ZDT4	best	**8,24E-05**	7,19E-01	5,01E-01	2,56E+00
	worst	**7,47E-05**	2,89E-02	6,93E-02	2,03E-01
	mean	**7,74E-05**	2,88E-01	1,94E-01	1,16E+00
	median	**7,76E-05**	2,34E-01	1,74E-01	9,38E-01
	std	**1,67E-06**	2,09E-01	9,52E-02	6,88E-01
ZDT6	best	**6,61E-05**	2,99E-03	4,21E-03	1,69E-01
	worst	**6,13E-05**	2,88E-04	6,68E-04	3,52E-04
	mean	**6,34E-05**	1,45E-03	2,05E-03	3,35E-02
	median	**6,34E-05**	1,87E-03	1,74E-03	3,35E-03
	std	**1,12E-06**	8,96E-04	1,00E-03	4,86E-02

Best results are marked in bold

The table I presents compelling evidence of the superior performance of our proposed method across a range of test problems, namely ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6, as assessed by the IGD metric. Notably, our MOSTOA demonstrates rapid convergence to the Pareto set in scenarios where ZDT1 exhibits a convex front, and ZDT2 exhibits a non-convex front (See Fig.2 and Fig .3).

TABLE II. COMPARATIVE RESULTS OF HV METRIC

		MOSTOA	MOGWO	MSSA	MOEAD
ZDT1	best	**7,22E-01**	7,09E-01	5,96E-01	7,11E-01
	worst	**7,22E-01**	6,48E-01	4,83E-01	5,90E-01
	mean	**7,22E-01**	6,93E-01	5,48E-01	6,85E-01
	median	**7,22E-01**	6,96E-01	5,50E-01	7,05E-01
	std	**9,12E-05**	1,09E-02	2,58E-02	3,73E-02
ZDT2	best	**4,46E-01**	4,23E-01	3,13E-01	4,38E-01
	worst	**4,45E-01**	9,09E-02	1,91E-01	0,00E+00
	mean	**4,46E-01**	3,39E-01	2,61E-01	7,52E-02
	median	**4,46E-01**	4,10E-01	2,65E-01	2,06E-02
	std	**2,29E-04**	1,37E-01	2,59E-02	1,04E-01
ZDT3	best	**5,85E-01**	6,00E-01	6,46E-01	5,80E-01
	worst	**5,82E-01**	5,48E-01	4,70E-01	1,30E-01
	mean	**5,84E-01**	5,81E-01	5,55E-01	4,22E-01
	median	**5,84E-01**	5,81E-01	5,51E-01	4,18E-01
	std	**5,16E-04**	1,07E-02	4,57E-02	8,87E-02
ZDT4	best	**7,22E-01**	9,09E-02	0,00E+00	0,00E+00
	worst	**7,22E-01**	0,00E+00	0,00E+00	0,00E+00
	mean	**7,22E-01**	5,86E-03	0,00E+00	0,00E+00
	median	**7,22E-01**	0,00E+00	0,00E+00	0,00E+00
	std	**2,71E-05**	2,27E-02	0,00E+00	0,00E+00
ZDT6	best	**3,90E-01**	3,85E-01	3,76E-01	3,83E-01
	worst	**3,90E-01**	3,26E-01	2,52E-01	0,00E+00
	mean	**3,90E-01**	3,59E-01	3,38E-01	2,08E-01
	median	**3,90E-01**	3,50E-01	3,50E-01	2,81E-01
	std	**2,67E-05**	1,95E-02	3,19E-02	1,76E-01

Best results are marked in bold

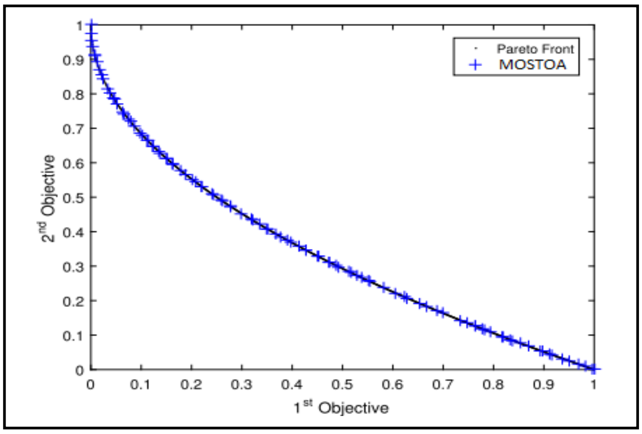

Fig.2. Pareto Front of the MOTOA algorithm for ZDT1

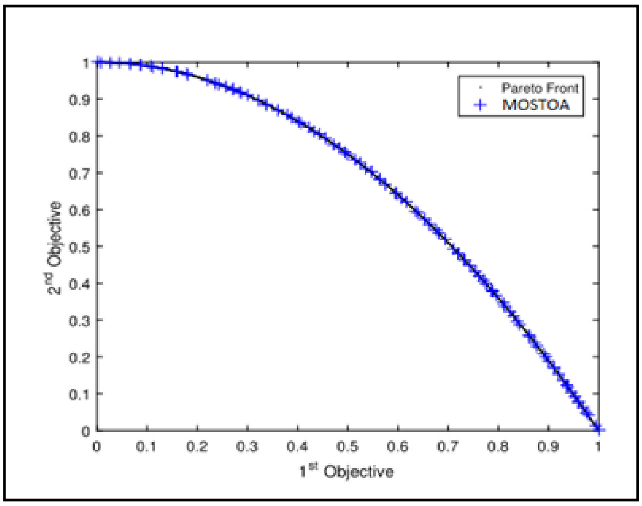

Fig.3. Pareto Front of the MOTOA algorithm for ZDT2

In accordance with the Hypervolume (HV) metric, the results presented in Table II provide strong statistical evidence of the superior performance of the MOSTOA algorithm when compared to other comparative methods across all test cases. Consequently, the proposed MOSTOA algorithm stands out as the top-performing approach among those examined. Visual representations in Figures 2-3 further illustrate that the non-dominated solutions generated by the MOSTOA algorithm exhibit a well-distributed alignment with the true Pareto front.

V. CONCLUSION

In our research, we have implemented three key strategies aimed at striking an optimal balance between exploration and exploitation attributes to extend the conventional Single-Objective Optimization Algorithm (STOA) into its Multi-Objective counterpart, known as MOSTOA. Firstly, we incorporate a population archive, serving as a dedicated repository for storing and retrieving non-dominated solutions acquired throughout the optimization process. Secondly, we employ a leader's solution to guide the primary population towards promising areas within the search space. Lastly, we utilize a density estimator, specifically based on the Grid Adaptive Strategy, to ensure comprehensive coverage of the search space. The comparative results, assessed using established metrics within the multi-objective optimization domain, have conclusively demonstrated the effectiveness of our proposed MOSTOA. It exhibits superior convergence and diversity when compared to alternative algorithms.

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