Finitely Infinite – a Mathematician's Odyssey

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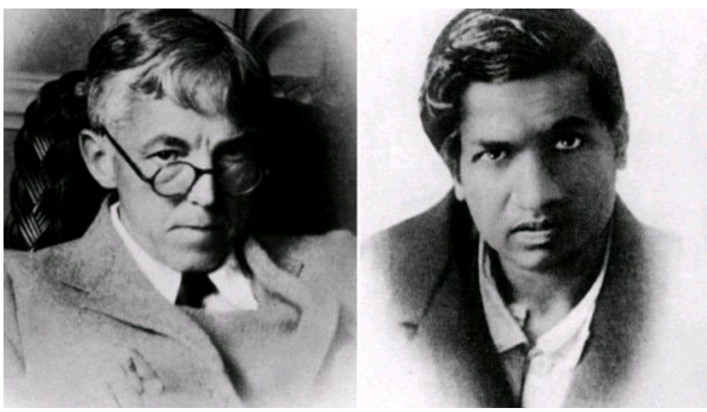
শ্রীনিবাস রামানুজন

"Touching the infinity with Ramanujan" where his 3 Proofs are shown in details: Infinite Sum, Infinity Roots, Taxi-Cab 'Hardy-Ramanujan' Number with letters exchanged between Hardy and Ramanujan along with Hardy's remarks of appreciation for Ramanujan with a "Brief History of discovering Ramanujan's Genius" by Hardy. Covering the aspects about Ramanujan's Brilliance' as any when expressed by Hardy to his fellow colleagues over several discussions.



Left – Hardy. *Right* – Ramanujan

[*Hardy remarked about Ramanujan as 'The man who taught infinity'*

On about January 31, 1913 a mathematician named G. H. Hardy in Cambridge, England received a package of papers with a cover letter that began: "Dear Sir, I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age…." and went on to say that its author had made "startling" progress on a theory of divergent series in mathematics, and had all but solved the longstanding problem of the distribution of prime numbers. The cover letter ended: "Being poor, if you are convinced that there is anything of value I would like to have my theorems published…. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for the trouble I give you. I remain, Dear Sir, Yours truly, S. Ramanujan".

What followed were at least 11 pages of technical results from a range of areas of mathematics.] *Excerpts from"* [1 - 6]

[G. H Hardy and J.E. Littlewood were two giants of mathematics in the first half of the 20th century, especially in number theory and analysis. After dinner in Trinity one evening, Hardy mentioned to Littlewood some of the claims he had received in the mail from an unknown Indian. Some assertions they knew well, others they could prove, others they could disprove, but many they found not only fascinating and unusual but also impossible to resolve.

Bertrand Russell wrote that by the next day he "found Hardy and Littlewood in a state of wild excitement because they believe they have found a second Newton, a Hindu clerk in Madras making 20 pounds a year."

Hardy quoted "... On the other hand there were things of which it was impossible that he would remain in ignorance … so I had to try to teach him, and in a measure I succeeded, though I obviously learnt from him much more than he learnt from me." Hardy even compared Ramanujan's brilliance with Bernoulli and Euler. Neville's stated "the discovery of the genius of S. Ramanujan of Madras promises to be the most interesting event of our time in the mathematical world …"

Ramanujan was invited and on 17 March 1914 he traveled to England by ship leaving his wife to stay with his parents in India. During his short life, Ramanujan independently compiled nearly 3,900 results. Many were completely novel; his original and highly unconventional, like: the Ramanujan prime, the Ramanujan theta function, partition formula and mock theta functions, which opened entire new areas of work and inspired a vast amount of further research. Hardy found these results "much more intriguing" than Gauss's work on integrals.

Ramanujan once said, "An equation for me has no meaning unless it expresses a thought of God." Sadly, Ramanujan left this world at the age of just 32, on April 26 1920.] *Excerpts from"* [1 - 6]

Letter from S. Ramanujan to G.H. Hardy (16 January 1913)

Dear Sir,

I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age. I have had no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as "startling".

Just as in elementary mathematics you give a meaning to a^n *when n is negative and fractional to conform to the law which holds when n is a positive integer, similarly the whole of my investigations proceed on giving a meaning to Eulerian Second Integral for all values of n. My friends who have gone through the regular course of University education tell me that* [eq. 1] *is true only when n is positive. They say that this integral relation is not true when n is negative. Supposing this is true only for positive values of n and also supposing the definition* [eq. 2] *to be universally true, I have given meanings to these integrals and under the conditions I state the integral is true for all values of n negative and fractional. My whole investigations are based upon this and I have been developing this to a remarkable extent so much so that the local mathematicians are not able to understand me in my higher flights.*

Very recently I came across a tract published by you styled Orders of Infinity in page 36 of which I find a statement that no definite expression has been as yet found for the number of prime numbers less than any given number. I have found an expression which very nearly approximates to the real result, the error being negligible. I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for the trouble I give you.

I remain, Dear Sir, Yours truly,
S. Ramanujan

P.S. My address is S.Ramanujan, Clerk Accounts Department, Port Turst, Madras, India

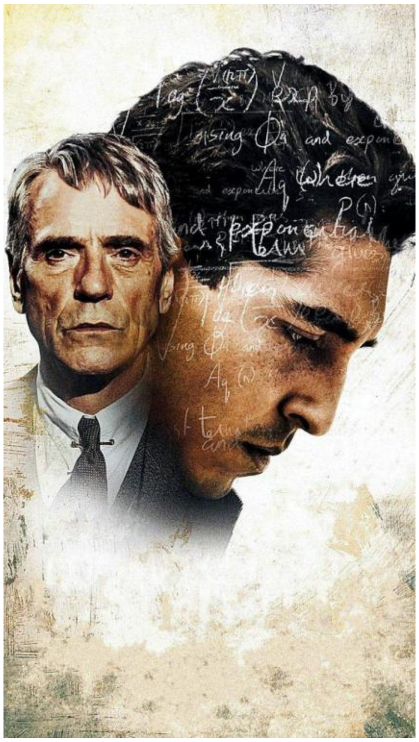
(1)

Madras Port Trust Office
Accounts Department.
27th February 1913.

Dear Sir,

I am very much gratified on perusing your letter of the 8th February 1913. I was expecting a reply from you similar to the one which a Mathematics Professor at London wrote asking me to study carefully Bromwich's Infinite series and not fall into the pitfalls of divergent series. I have found a friend in you who views my labours sympathetically. This is already some encouragement to me to proceed with my onward course. I find in many a place in your letter rigorous proofs are required and so on and you ask me to communicate the methods of proof. If I had given you my methods of proof I am sure you will follow the London Professor. But as a fact I did not give him any proof but made some assertions as the following under my new theory. I told him that the sum of an infinite no of terms of the series:— $1+2+3+4+\cdots = -1/12$ under my theory. If I tell you this you will at once point out to me the lunatic asylum as my goal. I dilate on this simply to convince you that you will not be able to follow my methods of proof if I indicate the lines on which I proceed in a single letter. You may ask how you can accept results based upon wrong premises. What I tell you is this. Verify the results I give and if they agree with your results, got by treading on the groove in which the present day mathematicians move, you should at least grant that there may be some truths in my fundamental basis. So what I now want at this stage is for eminent professors like you to recognise that there

Letter exchanged between Ramanujan and Hardy. *From [4, 5, 6]*



Taxi Cab Number or Hardy-Ramanujan Number

Indian Mathematician Ramanujan was ill at hospital and a Cambridge mathematician Hardy visited him to see in hospital. On a informal talking... He said to Ramanujan that "the taxi by which i came had a boring number 1729".... Ramanujan said "No. Hardy... It’s the smallest number that can be represented as the sum of two cubes in two different ways"....

$$1729 = 9^3 + 10^3 = 729 + 1000 = 1729$$

$$1729 = 12^3 + 1^3 = 1728 + 1 = 1729$$

Michio Kaku on Ramanujan — “*Srinivasa Ramanujan was the strangest man in all of mathematics, probably in the entire history of science. He has been compared to a bursting supernova, illuminating the darkest, most profound corners of mathematics, before being tragically struck down by tuberculosis at the age of 33… Working in total isolation from the main currents of his field, he was able to rederive 100 years’ worth of Western mathematics on his own. The tragedy of his life is that much of his work was wasted rediscovering known mathematics.*” *From [10]*

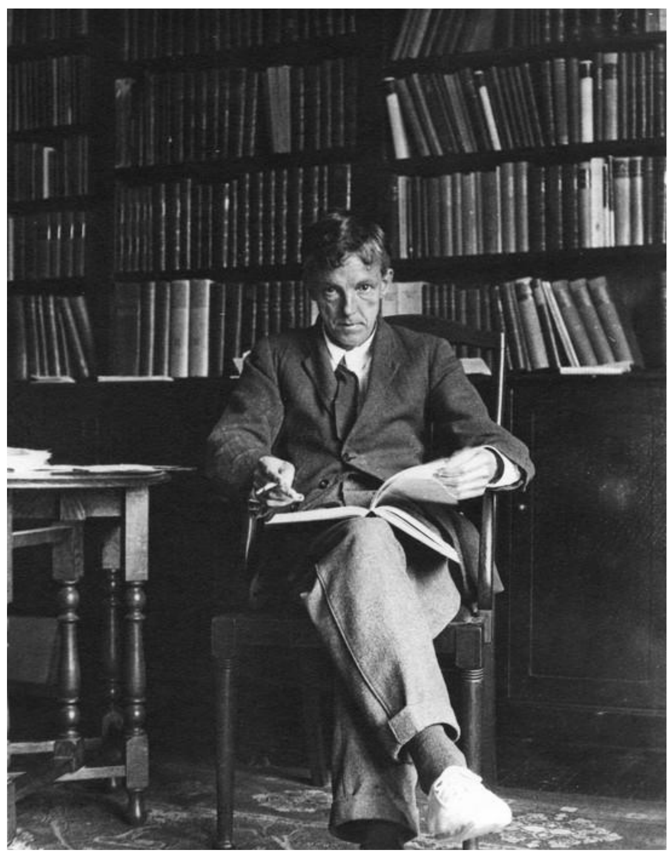
[In 1903, when Ramanujan was 15 years old, he obtained a copy of a book called “Synopsis of Elementary Results in Pure and Applied Mathematics”. It was written by G. S. Carr in 1886. This book contains a compilation of about 5000 theorems.... This book is considered to have awakened the genius.] *Excerpts from [10]*

The Man Who Knew Infinity (2016). *Warner Bros. From [7, 8]*

The root tends to infinity having answer is 3

$$3$$
$$= \sqrt{9}$$
$$= \sqrt{1+8}$$
$$= \sqrt{1+2*4}$$
$$= \sqrt{1+2\sqrt{16}}$$
$$= \sqrt{1+2\sqrt{1+15}}$$
$$= \sqrt{1+2\sqrt{1+3*5}}$$
$$= \sqrt{1+2\sqrt{1+3\sqrt{25}}}$$
$$= \sqrt{1+2\sqrt{1+3\sqrt{1+24}}}$$
$$= \sqrt{1+2\sqrt{1+3\sqrt{1+4*6}}}$$
$$= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{36}}}}$$
$$= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+35}}}}$$
$$= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5*7}}}}$$
$$= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{49}\ldots\ldots\infty}}}}$$

Excerpts from [9]



Godfrey Harold Hardy — The mathematician who discovered the genius Ramanujan.

***What is* $\mathbf{1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16+17\ldots\ldots\ldots\ldots\ldots\infty}$?**[†]

The answer is $-1/12$[‡]

Considering three Series of *Real Numbers* – $S^1,\ S^2,\ S^3$

$$S^1 = 1-1+1-1+1-1+1-1+1-1+1-1\ldots\ldots\infty$$

By grouping,

$$1+(-1+1)+(-1+1)+(-1+1)+(-1+1)\ldots..\infty\ = 1$$

Subtracting 1 from S^1,

$$1-S^1 = 1-(1-1+1-1+1-1+1-1+1-1+1\ldots\ldots\infty)$$

Or, $1-S^1 = S^1$ ∵ by subtracting 1 the series remains same and is equal to S^1
Or, $2S^1 = 1$
Or, $S^1 = 1/2$

$$S^2 = 1-2+3-4+5-6+7-8+9\ldots\ldots\infty$$

Taking again S^2 but by taking a 0 in front,

$$S^2 = 0+1-2+3-4+5-6+7-8+9\ldots\ldots\infty$$

Adding them together,

$2S^2 = 1-1+1-1+1-1+1-1\ldots\ldots\infty \mid RHS =\ S^1$

$So, 2S^2 = S^1\ (S^1 = 1/2)$
$So, 2S^2 = 1/2$
$So, S^2 = 1/4$

$$S^3 = 1+2+3+4+5+6+7+8+9\ldots\ldots\infty$$

$$S^2 = 1-2+3-4+5-6+7-8+9\ldots\ldots\infty$$

$$S^3 - S^2 = (1+2+3+4+5+6+7+8+9\ldots\ldots\infty)-(1-2+3-4+5-6+7-8+9\ldots\ldots\infty)$$

$$Or, 1+2+3+4+5+6+7+8+9\ldots\ldots\infty\ -1+2-3+4-5+6-7+8-9\ldots\ldots\infty$$

$$Or, 0+4+0+8+0+12+0+16\ldots\ldots\infty$$

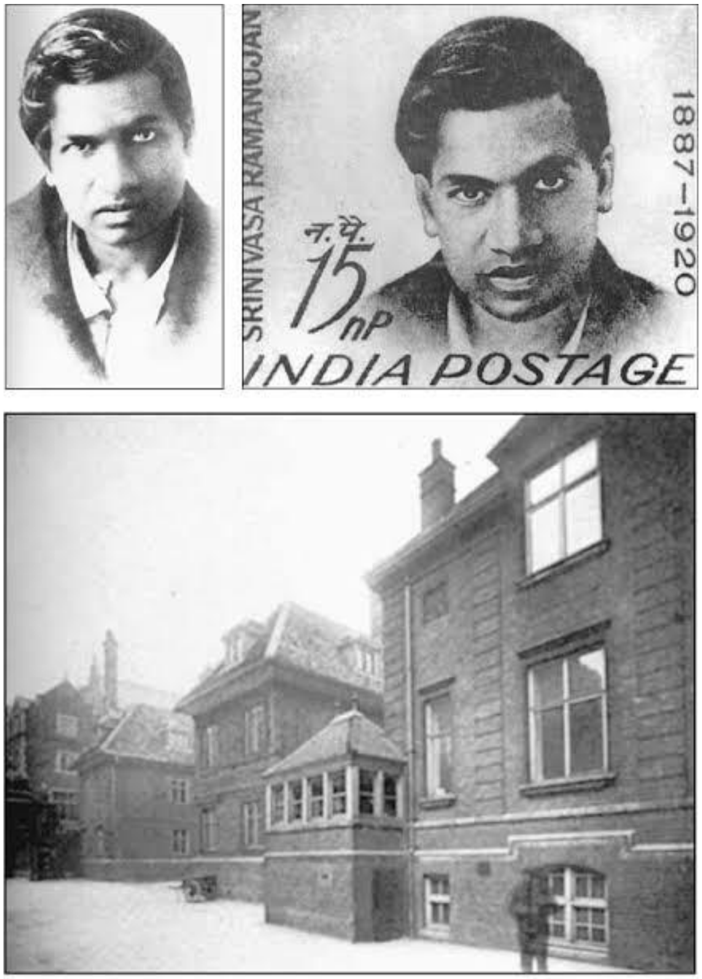
$$(Excluding\ 0)$$

$$We\ Get -\ 4+8+12+16\ldots\ldots\infty$$
$$Or, 4(1+2+3+4\ldots\ldots\infty\ )$$
$$Or, 4S^3$$

$$By\ writing\ in\ equation\ form\ldots.$$

$$S^3 - S^2 = 4S^3$$
$$Or, S^3 - 1/4 = 4S^3$$
$$Or, 4S^3 - S^3 = -1/4$$
$$Or, 3S^3 = -1/4$$
$$Or, S^3 = -(1/12)^*$$

‡†* There's been several deductions stating others forms of proofs while some approaches to further showing the usage of fundamental mathematics is not trivial as used for this proof. Those are not been concerned [*neither denied nor opinioned*] as this paper is subject to show the works of Ramanujan.



TOP LEFT: Srinivasa Ramanujan, after he had grown his hair and cut it in the European style.
TOP RIGHT: A postage stamp honoring Ramanujan.
BOTTOM: Bishop's Hall in Cambridge. Ramanujan lived in the building on the right from 1915 to 1917.

From [11]

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