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Anwar El Fadil El Idrissi and Abdelhak Yaacoubi

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Anwar EL FADIL EL IDRISSI[1,2*] and Abdelhak YAACOUBI[2,1†]

[1*]MAEGE, Hassan II University of Casablanca-AIN SEBAA, Casablanca, Morocco, .
[2]MAEGE, Hassan II University of Casablanca-AIN SEBAA, Casablanca, Morocco, .

*Corresponding author(s). E-mail(s): anwarelfadilelidrissi@gmail.com;
Contributing authors: abdelhak.yaacoubi@etu.univh2c.ma ;
†These authors contributed equally to this work.

Abstract

Public conversations are increasingly taking place on social networks due to the emergence of new communication channels. These public transactional spaces are deeply embedded in our culture and, at the same time, are sensitive sensors of human behavior and collective emotions. As a result, the transmission of ideas is relatively fast. It can be used for good and evil and poses one of the greatest threats to society as it can disrupt financial, political, and economic markets. Several mathematical models based on epidemiological models have been constructed to understand this complex transmission process, which is mainly influenced by psychosocial factors. In this study, we propose a new paradigm for the diffusion of ideas that accounts for various variations in social network membership categories. With this new model, we describe the allowed equilibrium state, the primary conditions for its stability, and the stability parameters of the model.

Keywords:
Human Behavior, transmission, epidemiological, sociopsychological, stability

1 Introduction

Indirectly or directly, ideas are transmitted, analogous to the spread of disease. This phenomenon evolves as it acquires momentum and generates new dimensions and hues of interest. It has been known to adapt to various environments, enhancing its allure and expanding its appeal. An idea's propagation and control can be as unpredictable as cancer, with the potential to transcend its originator. When administered effectively, an idea with contagious influence can become a potent change agent and alter the course of history.

An "epidemic" is commonly defined as the transition from susceptibility to communicability. This transformation occurs when exposed to certain factors, typically infectious substances. Although the word "epidemic" is often used to refer to an infectious disease epidemic, it can also have metaphorical connotations. It needs clarification on the many motives that make up this process, including susceptible, intermediate, infectious agents, and other variables.

Nevertheless, once individuals are susceptible to a particular idea, they can disseminate it to others. As individuals compromise their critical thinking skills and disseminate misinformation, this trend may result in an intellectual "epidemic".

Communication is the utilization of information to accomplish a desired result. The outcome of this information can be used to evaluate its accuracy. Regardless of their composition, all communication processes share common characteristics. For example, Goffman and Newill (1964) demonstrated that the factors contributing to the proliferation of infectious diseases and ideas are identical. Given the two most common examples of the communication process—the dissemination of ideas and the transmission of infectious diseases—it is perhaps not surprising. Therefore, the propagation of an epidemic is a viable method for studying the diffusion process. In essence, the principles of epidemics can be applied to analyzing any expanding process, not just the spread of infectious diseases.

The propagation of (ideas, rumors, epidemics...) is influenced by psychological variables, and mathematical models based on epidemics have been devised to understand this complicated process better. Including, but not limited to, the works of Daley and Kendall (1964), Daley and Kendall (1965), Dietz (1967), Maki (1973), Rapoport (1953), Rapoport et al. (1953), and Rapoport and Rebhun (1952).

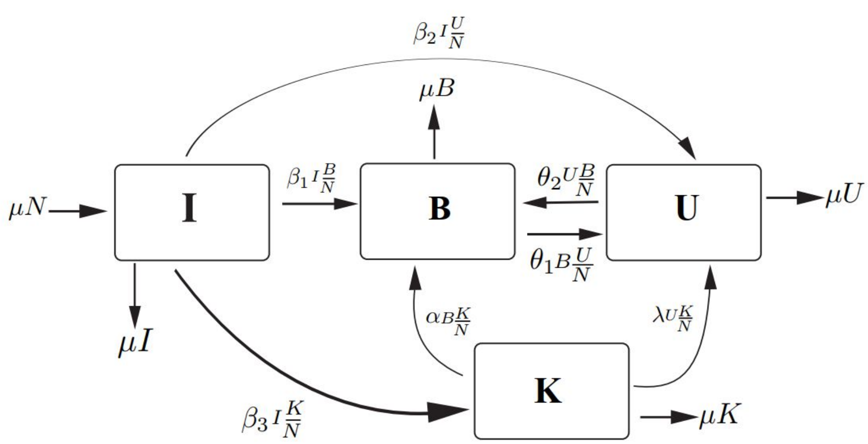
Regarding the spreading of specific ideas or information, in the "IBUK" model, which outlines the behaviors of individuals who are distributed within a network. These individuals can be classified as ignorant, believers, unbelievers, or Knowledgeable about the idea. Once an idea emerges among the population, each individual will act based on their beliefs. Believers will spread the idea to their neighbors, while other classifications will respond differently. Those who don't accept the concept won't be spreading it, whereas the informed group those who heard about it but didn't recall - won't be actively promoting it.

Thoroughly inspired by Misra's (2012) mathematical model for the spread of political parties, we have constructed a dynamic model which applies to ideas. However, our population is segmented into four subcategories instead of Misra's (2012) three, which complicates our computations slightly. As a result, the use of Maple is sometimes necessary.

2 Mathematical Model

Let N be the total number of persons and the rate at which an individual enters or leaves the social network. This population is divided into four groups: ignorants, believers, unbelievers, and knowledgeable. In the following, we denote by I the number of ignorants, B the number of believers, U the number of unbelievers, and K the number of knowledgeable. We assume that an ignorant can become a believer, an unbeliever, or a knowledgeable with rates β_1, β_2 or β_3, respectively. A believer can become an unbeliever or a knowledgeable with rate θ_1 or λ, respectively. Moreover, an unbeliever can become a believer with a rate of θ_2 Note that a knowledgeable can change class to an unbeliever with rate α respectively, and leave the social network, as all the individuals of the four other groups, with rate μ.

All these transmission rules are synthesized in the following diagram



and can be written using the mathematical system of ordinary differential equations

$$\begin{cases} \frac{dI}{dt}(t) = \mu N - \beta_1 \frac{I(t)B(t)}{N} - \beta_2 \frac{I(t)U(t)}{N} - \beta_3 \frac{I(t)K(t)}{N} - \mu I(t) \\ \frac{dB}{dt}(t) = \beta_1 \frac{I(t)B(t)}{N} - \theta_1 \frac{B(t)U(t)}{N} + \alpha \frac{B(t)K(t)}{N} + \theta_2 \frac{U(t)B(t)}{N} - \mu B(t), \\ \frac{dU}{dt}(t) = \beta_2 \frac{I(t)U(t)}{N} + \theta_1 \frac{B(t)U(t)}{N} - \theta_2 \frac{U(t)B(t)}{N} + \lambda \frac{U(t)K(t)}{N} - \mu U(t), \\ \frac{dK}{dt}(t) = \beta_3 \frac{I(t)K(t)}{N} - \alpha \frac{B(t)K(t)}{N} - \lambda \frac{U(t)K(t)}{N} - \mu K(t), \end{cases} \tag{1}$$

$$\begin{cases} \frac{di}{dt}(t) = \mu - \beta_1 i(t)b(t) - \beta_2 i(t)u(t) - \beta_3 i(t)k(t) - \mu i(t), \\ \frac{db}{dt}(t) = \beta_1 i(t)b(t) - \theta b(t)u(t) + \alpha b(t)k(t) - \mu b(t), \\ \frac{du}{dt}(t) = \beta_2 i(t)u(t) + \theta b(t)u(t) + \lambda u(t)k(t) - \mu u(t), \\ \frac{dk}{dt}(t) = \beta_3 i(t)k(t) - \alpha b(t)k(t) - \lambda u(t)k(t) - \mu k(t). \end{cases} \tag{2}$$

with $\mu, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2, \alpha$ and λ strictly non negative real numbers. By setting $\theta = \theta_1 - \theta_2$ and $i = \frac{I}{N}, b = \frac{B}{N}, u = \frac{U}{N}$ and $k = \frac{K}{N}$, one obtains Moreover, $I + B + U + K = N$ so $i + b + u + k = 1$ and we can only study the following system of three ordinary differential equations

$$\begin{cases} \frac{db}{dt}(t) = \beta_1(1 - b(t) - u(t) - k(t))b(t) - \theta b(t)u(t) + \alpha b(t)k(t) - \mu b(t), \\ \frac{du}{dt}(t) = \beta_2(1 - b(t) - u(t) - k(t))u(t) + \theta b(t)u(t) + \lambda u(t)k(t) - \mu u(t), \\ \frac{dk}{dt}(t) = \beta_3(1 - b(t) - u(t) - k(t))k(t) - \alpha b(t)k(t) - \lambda u(t)k(t) - \mu k(t), \end{cases} \tag{3}$$

and obtain i with $i(t) = 1 - b(t) - u(t) - k(t)$. In the following, we assume that $\theta > 0$ but the study will be the same in the contrary case.

3 Equilibrium analysis

The study of equilibrium states leads us to solve the following system of three equations

$$\begin{cases} \beta_1(1 - b(t) - u(t) - k(t))b(t) - \theta b(t)u(t) + \alpha b(t)k(t) - \mu b(t) = 0, \\ \beta_2(1 - b(t) - u(t) - k(t))u(t) + \theta b(t)u(t) + \lambda u(t)k(t) - \mu u(t) = 0, \\ \beta_3(1 - b(t) - u(t) - k(t))k(t) - \alpha b(t)k(t) - \lambda k(t)u(t) - \mu k(t) = 0, \end{cases} \tag{4}$$

Keeping in mind that an acceptable equilibrium state is a triplet of non-negative solutions to these three equations, (b, u, k). The aforementioned problem was solved using Maple, and eight solutions were obtained. While some answers are simple to acquire, not all are. The answer is expressed as a triplet, (b, u, k). State is the first one that stands out.

$E_1 = (0,0,0)$. The three following solutions are

$$E_2 = \left(\frac{\beta_1 - \mu}{\beta_1}, 0, 0\right), E_3 = \left(0, \frac{\beta_2 - \mu}{\beta_2}, 0\right), E_4 = \left(0, 0, \frac{\beta_3 - \mu}{\beta_3}\right),$$

which, if and only if, are acceptable equilibrium states.

$$\beta_1 > \mu, \beta_2 > \mu \text{ and } \beta_3 > \mu, \tag{5}$$

respectively. Another possible answer is

$$E_5 = \left(\frac{\mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu)}{\theta(\theta + \beta_1 - \beta_2)}, \frac{\theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2)}{\theta(\theta + \beta_1 - \beta_2)}, 0\right),$$

which is admissible if

$$\begin{cases} \theta + \beta_1 - \beta_2 > 0, \\ \mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu) > 0, \\ \theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2) > 0, \end{cases} \text{ or } \begin{cases} \theta + \beta_1 - \beta_2 < 0, \\ \mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu) < 0, \\ \theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2) < 0 \end{cases} \tag{6}$$

Let us remark that, if

$$\begin{cases} \theta + \beta_1 - \beta_2 < 0 \\ \mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu) < 0 \\ \theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2) < 0 \end{cases}$$

then $\beta_1 < \mu$ and this is not compatible with the condition of admissibility of E_2. For this reason, we say that E_5 is admissible if and only if

$$\begin{cases} \theta + \beta_1 - \beta_2 > 0 \\ \mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu) > 0 \\ \theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2) > 0 \end{cases} \tag{7}$$

Similarly, the solutions.

$$E_6 = \left(\frac{\alpha(\beta_3 - \mu) - \mu(\beta_3 - \beta_1)}{\alpha(\alpha + \beta_3 - \beta_1)}, 0, \frac{\mu(\beta_3 - \beta_1) - \alpha(\beta_1 - \mu)}{\alpha(\alpha + \beta_3 - \beta_1)}\right)$$

and

$$E_7 = \left(0, \frac{\lambda(\beta_3 - \mu) - \mu(\beta_3 - \beta_2)}{\lambda(\lambda + \beta_3 - \beta_2)}, \frac{\mu(\beta_3 - \beta_2) - \lambda(\beta_2 - \mu)}{\lambda(\lambda + \beta_3 - \beta_2)}\right)$$

are admissible if and only if

$$\begin{cases} \alpha + \beta_3 - \beta_1 > 0, \\ \alpha\left(\beta_3 - \mu\right) - \mu\left(\beta_3 - \beta_1\right) > 0, \\ \mu\left(\beta_3 - \beta_1\right) - \alpha\left(\beta_1 - \mu\right) > 0, \end{cases} \text{ or } \begin{cases} \alpha + \beta_3 - \beta_1 < 0, \\ \alpha\left(\beta_3 - \mu\right) - \mu\left(\beta_3 - \beta_1\right) < 0, \\ \mu\left(\beta_3 - \beta_1\right) - \alpha\left(\beta_1 - \mu\right) < 0, \end{cases} \tag{8}$$

and

$$\begin{cases} \lambda + \beta_3 - \beta_2 > 0, \\ \lambda\left(\beta_3 - \mu\right) - \mu\left(\beta_3 - \beta_2\right) > 0, \\ \mu\left(\beta_3 - \beta_2\right) - \lambda\left(\beta_2 - \mu\right) > 0, \end{cases} \text{ or } \begin{cases} \lambda + \beta_3 - \beta_2 < 0, \\ \lambda\left(\beta_3 - \mu\right) - \mu\left(\beta_3 - \beta_2\right) < 0, \\ \mu\left(\beta_3 - \beta_2\right) - \lambda\left(\beta_2 - \mu\right) < 0, \end{cases} \tag{9}$$

respectively. By compatibility with the conditions (5), taken for the admissibility of E_2 and E_3, we can say that E_6 and E_7 are admissible if and only if

$$\begin{cases} \alpha + \beta_3 - \beta_1 > 0, \\ \alpha\left(\beta_3 - \mu\right) - \mu\left(\beta_3 - \beta_1\right) > 0, \\ \mu\left(\beta_3 - \beta_1\right) - \alpha\left(\beta_1 - \mu\right) > 0, \end{cases} \text{ and } \begin{cases} \lambda + \beta_3 - \beta_2 > 0, \\ \lambda\left(\beta_3 - \mu\right) - \mu\left(\beta_3 - \beta_2\right) > 0, \\ \mu\left(\beta_3 - \beta_2\right) - \lambda\left(\beta_2 - \mu\right) > 0, \end{cases} \tag{10}$$

respectively. For the last solution E_8, one has

$$\begin{cases} b = \frac{\mu(\alpha+\theta-\lambda)(\lambda+\beta_3-\beta_2)-\lambda(\alpha\beta_2+\theta\beta_3-\lambda\beta_1)}{(\alpha+\theta-\lambda)(\alpha\beta_2+\theta\beta_3-\lambda\beta_1)}, \\ u = \frac{\alpha(\alpha\beta_2+\theta\beta_3-\lambda\beta_1)-\mu(\alpha+\theta-\lambda)(\alpha+\beta_3-\beta_1)}{(\alpha+\theta-\lambda)(\alpha\beta_2+\theta\beta_3-\lambda\beta_1)}, \\ k = \frac{\theta(\alpha\beta_2+\theta\beta_3-\lambda\beta_1)-\mu(\alpha+\theta-\lambda)(\theta+\beta_1-\beta_2)}{(\alpha+\theta-\lambda)(\alpha\beta_2+\theta\beta_3-\lambda\beta_1)}, \end{cases} \tag{11}$$

If $\alpha\beta_2 + \theta\beta_3 - \lambda\beta_1$ and $\alpha + \theta - \lambda$ have the same sign, the solution E_8 is admissible if and only if

$$\begin{cases} \mu(\alpha + \theta - \lambda)\left(\lambda + \beta_3 - \beta_2\right) > \lambda\left(\alpha\beta_2 + \theta\beta_3 - \lambda\beta_1\right) \\ \alpha\left(\alpha\beta_2 + \theta\beta_3 - \lambda\beta_1\right) > \mu(\alpha + \theta - \lambda)\left(\alpha + \beta_3 - \beta_1\right) \\ \theta\left(\alpha\beta_2 + \theta\beta_3 - \lambda\beta_1\right) > \mu(\alpha + \theta - \lambda)\left(\theta + \beta_1 - \beta_2\right), \end{cases} \tag{12}$$

4 Stability Analysis

Taking into account the hypotheses (5), (7), (10) and (12), we will now ascertain the character of all eight equilibrium states in order for them to be admissible. For this, the sign of the eigenvalues of the Jacobian matrix associated with System 3 and calculated at each equilibrium state must be known. Noting that it is possible not to presume all of these conditions (5), (7), (10) and (12), we will have less acceptable equilibrium states and less stable nodes in this case. Therefore, it is conceivable to evaluate various sign combinations for (5), (7), (10) and (12), but the reasoning will not fundamentally change.

The Jacobian matrix associated to system (3) is

$$J = \begin{pmatrix} \beta_1(1-2b-u-k) - \theta u + \alpha k - \mu, -b(\theta+\beta_1), b(\alpha-\beta_1) \\ u(\theta-\beta_2), \beta_2(1-b-2u-k) + \theta b + \lambda k - \mu, u(\lambda-\beta_2) \\ -k(\alpha+\beta_3), -k(\lambda+\beta_3), \beta_3(1-b-u-2k) - \lambda u - \alpha b - \mu \end{pmatrix}$$

Let us set J_i this Jacobian matrix calculated at the equilibrium state E_i, for $i = 1, \ldots, 8$. One has

$$J_1 = \begin{pmatrix} \beta_1 - \mu & 0 & 0 \\ 0 & \beta_2 - \mu & 0 \\ 0 & 0 & \beta_3 - \mu \end{pmatrix}$$

and the hypothesis done to have the admissibility of E_2, E_3 and E_4 imply that the eigenvalues of J_1 are all strictly non negative, which leads us to say the equilibrium state E_1 is an unstable node.

If $\beta_1 < \mu, \beta_2 < \mu$ and $\beta_3 < \mu$ then E_2, E_3 and E_4 are not admissible equilibrium states but E_1 is locally asymptotically stable. Moreover,

$$J_2 = \begin{pmatrix} \mu - \beta_1 & \frac{(\beta_1+\theta)(\mu-\beta_1)}{\beta_1} & \frac{(\beta_1-\alpha)(\mu-\beta_1)}{\beta_1} \\ 0 & \frac{\theta(\beta_1-\mu)-\mu(\beta_1-\beta_2)}{\beta_1} & 0 \\ 0 & 0 & \frac{\mu(\beta_3-\beta_1)-\alpha(\beta_1-\mu)}{\beta_1} \end{pmatrix}.$$

The eigenvalues of J_2 are $v_2^1 = \mu - \beta_1 < 0, v_2^2 = \frac{\theta(\beta_1-\mu)-\mu(\beta_1-\beta_2)}{\beta_1} > 0$ and $v_2^3 = \frac{\alpha(\beta_1-\mu)-\mu(\beta_1-\beta_3)}{\beta_1} > 0$ due to the conditions taken to have the admissibility of the equilibrium states E_2, E_5 and E_6 respectively. Consequently E_2 is unstable and it will be the same for the equilibrium states E_3 and E_4.

If $\theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2) < 0$ and $\alpha(\beta_1 - \mu) - \mu(\beta_1 - \beta_3) < 0$ with $\beta_1 - \beta_2 < 0$ and $\beta_1 - \beta_3 < 0$ then $\beta_1 - \mu < 0$ and E_2 is not an admissible equilibrium state. But if $\theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2) < 0$ and $\alpha(\beta_1 - \mu) - \mu(\beta_1 - \beta_3) < 0$ with $\beta_1 - \beta_2 > 0$ and $\beta_1 - \beta_3 > 0$ and if $\beta_1 - \mu > 0$ then E_5 and E_6 are not admissible equilibrium states but E_2 is a locally asymptotically stable one.

For the three Jacobian matrix J_5, J_6 and J_7, the reasoning will be the same for the three ones so we only give the details for J_6 for example. In fact J_6 is in the form

$$J_6 = \begin{pmatrix} \beta_1(1-2b-k) + \alpha k - \mu & -b(\theta+\beta_1) & b(\alpha-\beta_1) \\ 0 & \beta_2(1-b-k) + \theta b + \lambda k - \mu & 0 \\ -k(\alpha+\beta_3) & -k(\lambda+\beta_3) & \beta_3(1-b-2k) - \alpha b - \mu \end{pmatrix}$$

It is clear that one of the eigenvalues of J_6 is

$$v_6^1 = \beta_2(1-b-k) + \theta b + \lambda k - \mu$$

calculated with $b = \frac{\alpha(\beta_3-\mu)-\mu(\beta_3-\beta_1)}{\alpha(\alpha+\beta_3-\beta_1)}$ and $k = \frac{\mu(\beta_3-\beta_1)-\alpha(\beta_1-\mu)}{\alpha(\alpha+\beta_3-\beta_1)}$, which gives, after some computations,

$$v_6^1 = \frac{\alpha\left(\alpha\beta_2+\theta\beta_3-\lambda\beta_1\right)-\mu(\alpha+\theta-\lambda)\left(\alpha+\beta_3-\beta_1\right)}{\left(\alpha+\theta-\lambda\right)\left(\alpha\beta_2+\theta\beta_3-\lambda\beta_1\right)},$$

and this eigenvalue is strictly non negative due to the conditions done for the admissibility of E_6 and E_8. This is sufficient to claim that E_6 is unstable, and it is the same for E_5 and E_7.

We can discuss also here about different possibilities of signs for the quantities $\alpha\left(\alpha\beta_2+\theta\beta_3-\lambda\beta_1\right)-\mu(\alpha+\theta-\lambda)\left(\alpha+\beta_3-\beta_1\right)$ and $\alpha\left(\alpha+\beta_1-\beta_3\right)$ and find in some cases the non admissibility of the eight equilibrium states and the locally asymptotically stability of E_6 but the reasoning is the same as previously. It remains to determine the nature of the equilibrium point E_8. In fact,

$$J_8 = \begin{pmatrix} -\beta_1 b & -\left(\beta_1+\theta\right)b & \left(\alpha-\beta_1\right)b \\ \left(\theta-\beta_2\right)u & -\beta_2 u & \left(\lambda-\beta_2\right)u \\ -\left(\alpha+\beta_3\right)k & -\left(\lambda+\beta_3\right)k & -\beta_3 k \end{pmatrix}$$

with (b,u,k) defined in (11). Although we were able to utilize Mathematica to calculate the J_8 eigenvalues, we ultimately settled on using a method that randomly produces some values for the parameters $\beta_1,\beta_2,\beta_3,\alpha,\lambda,\theta$ and μ, and that meet all the constraints as mentioned above and so allow for all the equilibrium states E_i, for $i=1,\ldots,8$. Therefore, we identify six random values for these parameters that keep this model stable while meeting all requirements.

For example

$$\begin{cases} \mu = 0.273753, \\ \alpha = 0.24005, \\ \lambda = 0.191556, \\ \theta = 0.957926, \\ \beta_1 = 0.375526, \\ \beta_2 = 0.315954, \\ \beta_3 = 0.466138, \end{cases}$$

we can easily compute the eigenvalues of this particular

$$J_8 = \begin{pmatrix} -0.0065186866 & -0.02314714715 & -0.002351668161 \\ 0.02411193540 & -0.0118669700 & -0.004672285616 \\ -0.2354642411 & -0.2192942672 & -0.1554239375 \end{pmatrix}$$

and see that they are all negative, which leads us to claim that the equilibrium state E_8 is stable. Let us remark that with this choice of parameters, we find again the unstability of the seven first equilibrium points E_i, for $i=1,\ldots,7$

5 Discussion

Our study introduces a fresh mathematical concept for spreading ideas on social media platforms based on user class transition. Our model considers the mingling necessities of distinct groups of users, leading to a more all-encompassing comprehension of idea propagation. The following conversation will explore our research's potential limitations, consequences, and future prospects.

In studying how ideas spread through social networks, our model incorporates the different types of members involved. This enables us to analyze how specific groups, such as believers or nonbelievers, impact dissemination of these ideas. Our model captures the intricate transmission paths affected by psychosocial elements, providing a more detailed representation than epidemiological models currently do.

Our comprehension of the dynamics of idea diffusion in social networks has markedly advanced through one of our primary discoveries - the effective identification of the stability of equilibrium 8. This critical development is a sturdy basis for future research and lays the groundwork for substantial progress in comprehending the intricacies of idea transmission in forthcoming articles.

Implications of our model's stability parameters, primary conditions for stability, and permitted equilibrium state delve into the mechanisms driving idea diffusion in social networks. Suppose the components that lead to an idea's persistence in a network can be identified. In that case, policymakers and stakeholders can create tactics to champion the propagation of constructive ideas or combat the perils of disinformation.

Our investigation has made valuable contributions, yet some limitations must be considered. One such limitation is our simplification of user behavior, which assumes that group members react similarly to ideas. However, personal beliefs, social norms, and cognitive biases can all influence an individual's response. Future research should include these factors in the model to better capture the spread of ideas.

The clustering coefficient and degree distribution of a social network's topology can substantially affect the diffusion process, which our model fails to consider. To better understand the impact of network structure on the propagation of ideas, it would be crucial to investigate this function and bring modifications to our model in future research.

Interventions that foster positive idea diffusion or tackle misinformation's negative consequences could be developed by examining the propagation of disinformation, false news, and innovations categories. Our model's scope could be extended to understand the underlying mechanisms for the spread of these ideas.

Our presentation reveals a fresh mathematical model for diffusion ideas in social networks. Applying various membership categories in this model offers a more intricate grasp of the transmission processes and the factors that guarantee ideal network stability. Progress has been achieved by identifying the stability of equilibrium 8, thereby establishing a strong foundation for future research to advance our understanding of idea diffusion dynamics, bringing insight to diverse fields like public health, economics, and politics.

6 Conclusion

Our study yielded a fresh, innovative approach to deciphering the dissemination of ideas through social networks, factoring in the shifting tendencies of different segments of users. Our model accounts for the need for various factions to coexist while ensuring their sustained presence, effectively deepening our understanding of the spread of ideas.

Investigating the factors contributing to an individual's belief may yield exciting information regarding the sociological, economic, and psychological influences at play. Valuable insights and alternative perspectives might arise from studying the extinction of both nonbelievers and believers. It is clear that socio-psychological factors are significant in the spread of ideas, and this warrants further exploration in future research.

The transmission of an idea can be understood at its core, and we can use this knowledge to benefit multiple industries. One area where this strategy could be useful is in finance. By honing our skills in changing how ideas spread throughout society, we can change the trajectory of a concept, leading to long-lasting and beneficial results. This could include promoting goodwill or reducing the damage caused by negative influences.

Incorporating knowledgeable individuals into our proposed model of idea diffusion yields new insights into the intricate interactions between diverse user groups in social networks. On this premise, our next objective will be to investigate the function of knowledge-based talents in the IBUK model. We will investigate the factors that determine an individual's level of expertise and how this impacts their interactions with other network users. By obtaining a deeper comprehension of the underlying dynamics, we aim to refine and improve our model's predictive potential. We were particularly interested in determining how the influence of knowledgeable individuals varies according to their network connections and how their status as knowledgeable influences their likelihood of becoming believers or non-believers. By increasing our knowledge of the IBUK model and the function of the knowledgeable within it, we can obtain a more comprehensive understanding of the communication process and its potential impact on society.

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