Flexible Production Network Vs Guaranteed Supply Under Disruption

Yanying Jing and Qinyun Li

July 17, 2023

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1 Introduction

The fundamental challenge in managing a supply chain system is effectively matching supply or capacity with unpredictable demand. This is typically achieved through safety stock, flexible production, and excess capacity to facilitate service deliveries, Lobel and Xiao (2017). Among variable types of flexible manufacturing techniques (Jordan and Graves, 1995), process flexibility is the capability to reallocate resources for producing various types of products, Goyal and Netessine (2011). This study focuses on the flexible production network design, including the manufacturer's capacity allocation and the retailer's replenishment decision. It evaluates stock-out risks in the case of supply chain disruptions. To mitigate the disruptions, retailers establish clear and frequent communications with their contract manufacturers that offer greater certainty in supply; manufacturers enter into supply contracts with their customers in return for orders with higher predictability. We study a 'supply-commitment' contract in which the manufacturers allocate capacity for each retailer, and the retailers make capacity purchase commitments to the manufacturers based on the allocated capacity. Under this contract, enterprises adjust their decisions to maximise profit. Via game theory perspective, this paper discusses the important role of contracts in overall inventory management optimisation. Note that the manufacturers, despite competing for the retailer's procurement quantity, hold significant power in the relationship to function as Stackelberg leaders during contractual negotiations.

2 Literature review

Our research is closely related to the stream of literature on production flexibility. Flexibility, as defined by Upton (1994), refers to the ability of a system to respond or react to changes without incurring significant penalties in terms of time, effort, or cost. Bish et al. (2005) studied non-flexible, fixed-proportional, and full-flexible, three types of allocation policies in a two-plant, two-product capacitated order-up-to system and found that the capacity allocation policies strongly influence supply chains' profit. Durango-Cohen and Yano (2006) showed flexible capacity is a highly effective strategy for mitigating forecast error during the investment stage for companies managing short-life-cycle products, i.e. electronics. Chou et al. (2011) took Jordan and Graves' automobile production as an example and designed a flexible production system to better cope with fluctuating supply and demand. Lyu et al. (2019) considered a supply chain network with a flexible production system manufacturing multiple products and built an allocation policy to ensure the target level of each product is attainable.

In supply contracts that incorporate customers' forecasts or advance orders, the customer's commitments and payments are the primary distinguishing factors. Quantity flexibility contracts require customers to provide advanced forecasts and commit to a minimum purchase quantity, Tsay (1999). In return, the supplier commits to supplying at a specified level above the forecast. Eppen and Iyer (1997) illustrated a special form of the quantity flexibility contract, a backup agreement, which means the customer could choose to pay for a specified fraction of his order and the remaining portion at a later time. Kamrad and Siddique (2004) considered the producer's profit-maximisation problem and suppliers' reactions including supplier's performance and aversion to reward and risk. Their modelling approach to risk minimization parallels the portfolio optimization problem in a risk/return tradeoff sense, to mitigate risks that result from fluctuations in order levels over time.

3 Model

A base model describes players in the supply chain network negotiating the contract on the strategic level at the beginning of a year and handling orders periodically over the year. The second model considers inventory replenishment and production planning under periodic contract negotiation. In this paper, we only discuss the base model, and the second model will be explored in the future.

3.1 Contract negotiation

Consider a supply chain network with two competitive retailers (she: $i = 1, 2$) and two competitive manufacturers (he: $j = 3, 4$). This is a Stackelberg game in which each manufacturer moves first (quoting the capacity allocation), and the retailers follow by entering into the supply contract (before the demand is realized).

The sequence of the supply contract negotiation is as follows:
1) Retailer i publishes her predictions of the annual order quantity to manufacturer j, denoted as y_{ij}, based on the annual demand forecast D_i:

$$\begin{aligned} E[D_1] &= E[y_{13}] + E[y_{14}] \\ E[D_2] &= E[y_{23}] + E[y_{24}] \end{aligned} \tag{1}$$

2) For retailer i, she announces an ordering requirement y_{ij} to maximise her expected profits.

$$\max \pi_i = r_i D_i - \Sigma_j y_{ij} wa_{ij}, \tag{2}$$

where π is the profit, r is the retail price and wa_{ij} is the wholesale price the manufacturer j offers to the retailer i. We assume the price demand function is linearly dependent, z_i is the price upper bound of retailer i when the same market demand ($E[D_1] = E[D_2]$) (Adida and DeMiguel, 2011; Liu, 2012). In other words, z_i represents retailer i's market power ($z_i \geq 0$) that captures comparative advantage over her competitors due to certain factors including customer loyalty, locations and reputation (Chen and Roma, 2011; Cho and Tang, 2014). Since the linear inverse demand function

$$\begin{aligned} r_1 &= z_1 - (D_1 + D_2) \\ r_2 &= z_2 - (D_1 + D_2) \end{aligned} \tag{3}$$

we have,

$$\pi_1 = (z_1 - y_{13} - y_{14} - y_{23} - y_{24})(y_{13} + y_{14}) - (y_{13} wa_3 + y_{14} wa_4) \tag{4}$$

In the same way,

$$\pi_2 = (z_2 - y_{13} - y_{14} - y_{23} - y_{24})(y_{23} + y_{24}) - (y_{23}wa_3 + y_{24}wa_4) \tag{5}$$

The following need to be satisfied for retailers maximising profit:

$$\begin{aligned} \frac{\partial \pi_1}{\partial y_{13}} = \frac{\partial \pi_1}{\partial y_{14}} = 0 \\ \frac{\partial \pi_2}{\partial y_{23}} = \frac{\partial \pi_2}{\partial y_{24}} = 0 \end{aligned} \tag{6}$$

Solving Eq (6) under $D_i, y_{ij} \in \mathbb{R}^+$, we have

$$\begin{cases} y_{13} = \frac{wa_4 - 2wa_3}{3} \\ y_{14} = \frac{wa_3 - 2wa_4}{3} \\ y_{23} = \frac{wa_4 - 2wa_3}{3} \\ y_{24} = \frac{wa_3 - 2wa_4}{3} \end{cases} \tag{7}$$

According to (7), we need to find y^*_{ij}, hence the retail prices are

$$\begin{cases} r_1 = z_1 - y^*_{13} - y^*_{14} - y^*_{23} - y^*_{24} \\ r_2 = z_2 - y^*_{13} - y^*_{14} - y^*_{23} - y^*_{24} \end{cases} \tag{8}$$

3) When the manufacturer receives the predictive requirement, his objective is to find the optimal menu of the contract k_{ij}, wa_j that maximises his expected profits. Manufacturer j and retailer i negotiate a quantity k_{ij} and the wholesale price wa_{ij}. The k_{ij} in the contract is a commitment for both the manufacturer and the retailer. It is the manufacturer j's obligation to deliver k_{ij} units within the agreed lead time, as the retailer i secured such amount by acquiring k_{ij} of production capacity from the total production capacity K_j. For retailer i, k_{ij} is the order commitment that $y_{ij} \geq k_{ij}$.

The manufacturers' objective functions are,

$$\max \pi_j = (wa_j - ca_j)\Sigma_i y^*_{ij} + (K_j - y^*_{13} - y^*_{23})(wa_3 - ck_3) \tag{9}$$

There are two main constraints in the manufacturers' decision-making:

$$\begin{cases} y_{ij} \geq k_{ij} \\ \Sigma_i k_{ij} \leq K_j \end{cases} \tag{10}$$

Where, $y_{ij} \geq k_{ij}$ indicates that the production capacity secured by retailer i from manufacturer j shouldn't be less than the expected order quantity between the two players. $\Sigma_i k_{ij} \leq K_j$ guarantees that manufacturer j won't oversell his maximum production capacity.

- *Case I* considers manufacturer j's total capacity is sufficient to satisfy the orders from two retailers, $\Sigma_i y_{ij} \leq K_j$. The manufacturer's objective functions are rewritten as

$$\begin{aligned} \max \pi_3 = (wa_3 - ca_3)(y^*_{13} + y^*_{23}) \\ \max \pi_4 = (wa_4 - ca_4)(y^*_{14} + y^*_{24}) \end{aligned} \tag{11}$$

Subject to,

$$
\begin{cases}
k_{ij} \geq 0 \\
wa_j \geq ca_j \\
y_{13}^* \geq k_{13} \\
y_{14}^* \geq k_{14} \\
y_{23}^* \geq k_{23} \\
y_{24}^* \geq k_{24} \\
y_{13}^* + y_{23}^* \leq K_3 \\
y_{14}^* + y_{24}^* \leq K_4 \\
\Sigma_i k_{ij} \leq K_j
\end{cases}
\tag{12}
$$

To solve the nonlinear program problem (wa_j^*, k_{ij}^*) in (11) and (12), we apply the Karush–Kuhn–Tucker (KKT) conditions method, where the optimization problem is considered as,

$$
\begin{aligned}
\max \pi_j(wa_j, k_{ij}) \\
s.t. g_m(wa_j, k_{ij}) \geq 0 \\
h_n(wa_j, k_{ij}) = 0
\end{aligned}
\tag{13}
$$

Corresponding to the constrained optimization problem can form the Lagrangian function,

$$
\begin{aligned}
\max L_3(wa_3, k_{13}, k_{23}, \mu, \lambda) = \pi_3 + \mu_m g_m(wa_3, k_{13}, k_{23}) + \lambda_n h_n(wa_3, k_{13}, k_{23}) \\
\max L_4(wa_4, k_{14}, k_{24}, \mu, \lambda) = \pi_4 + \mu_m g_m(wa_4, k_{14}, k_{24}) + \lambda_n h_n(wa_4, k_{14}, k_{24})
\end{aligned}
\tag{14}
$$

We have the KKT conditions and then can solve the nonlinear program problem (wa_j^*, k_{ij}^*), $g_m(m = 1, ..., M)$ are the inequality constraint functions and $h_n(m = 1, ..., N)$ are the equality constraint functions, μ and λ are the corresponding KKT multipliers respectively. The numbers of inequalities and equalities are denoted by M and N respectively.

$$
\begin{cases}
\nabla \pi_j(wa_j^*, k_{ij}^*) + \sum_{m=1,...,M}^{M} \mu_m \nabla g_m(wa_j^*, k_{ij}^*) + \sum_{n=1,...,N}^{N} \lambda_n \nabla h_n(wa_j^*, k_{ij}^*) = 0 \\
\mu_m g_m(wa_j^*, k_{ij}^*) = 0 \\
\lambda_n h_n(wa_j^*, k_{ij}^*) = 0 \\
\lambda_n \geq 0 \\
g_m(wa_j^*, k_{ij}^*) \geq 0
\end{cases}
\tag{15}
$$

4) By substituting k_{ij}^* and wa_j^* into Eq (7) and Eq (8), the retailer's optimal order quantities and retail prices are obtained.

3.2 Inventory policy

Under the contract (wa_j, k_{ij}), retailers determine replenishment decisions (after demand is realized). We consider that the order-up-to (OUT) replenishment policy is applied at all echelons (Disney et al., 2013) because it is effective in minimizing inventory costs. Our model operates on a discrete-time periodic basis, making it relatively simple and reducing the complexity of our analysis, Karlin (1960).

Retailer i's replenishment decisions and manufacturer j's production quantity after realising the demand at time t are,

$$\begin{aligned} o_i &= tns_i + \sum_{l=1}^{L_i} \hat{d}_{i,t,t+l} - \sum_{l=1}^{L_i-1} \sum_j o_{ij,t-l} - ns_{i,t} \\ o_j &= tns_j + \sum_{l=1}^{L_j} \hat{d}_{j,t,t+l} - \sum_{l=1}^{L_j-1} o_{j,t-l} - ns_{j,t} \end{aligned} \tag{16}$$

Note, the commitment $y_{i,j}$ is an annual figure and can be explicitly split into a production capacity a retailer bought every certain period. For example, retailer i believes she will place orders n times in a year (based on her review period) and asks the manufacturer j to secure $y_{i,j}/n$ of the production capacity for her. $y_{i,j}/n$ is then related to the real order quantity at time t from retailer i to manufacturer j, $o_{ij,t}$. The quantity of $o_{ij,t}$ is also determined by the commitment between retailer i and the two manufacturers. Therefore, the following order decision applies:

$$o_{ij,t} = \max(o_{i,t}\frac{y_{ij}}{\Sigma_j y_{ij}}, \frac{y_{ij}}{n}) \tag{17}$$

L_i denotes retailer i's replenishment lead time including a unit review period. L_j is manufacturer j's production lead time (a unit review period inclusive). tns is a safety stock used to ensure a strategic level of inventory availability, and it is common to assume $tns_i = z_i^{score}\sigma_{ns_i}$. Here σ_{ns_i} is the standard deviation of the net stock levels. The net stock

$$\begin{aligned} ns_{i,t} &= ns_{i,t-1} + o_{i,t-L_i} - d_{i,t} \\ ns_{j,t} &= ns_{j,t-1} + o_{j,t-L_j} - d_{j,t},\ d_{j,t} = \Sigma_i o_{ij,t} \end{aligned} \tag{18}$$

complete the model, Li et al. (2014).
The retailers' replenishment decisions after retailers realize the demand d_i are,

$$\begin{aligned} o_1 &= tns_1 + d_{1,t,t+L_1} + \sum_{l=1}^{L_1-1} d_{1,t,t+l} - \sum_{l=1}^{L_1-1} (y_{13,t-l} + y_{14,t-l}) - ns_{1,t} \\ o_2 &= tns_2 + d_{2,t,t+L_2} + \sum_{l=1}^{L_2-1} d_{2,t,t+l} - \sum_{l=1}^{L_2-1} (y_{23,t-l} + y_{24,t-l}) - ns_{2,t} \end{aligned} \tag{19}$$

Retailer i's inventory cost ic_i includes holding costs $hc_i hs_i$ and backlog costs (stock-out costs) $bc_i bs_i$. The inventory on-hand $hc_i = \max(ns_{i,t}, 0)$ and the backlog $hc_i = \max(-ns_{i,t}, 0)$. hs_i and bc_i are unit holding and stock-out cost respectively.

Retailer i's inventory costs (ic_i) and total profits (Π_i) functions, as well as manufacturer j's inventory costs (ic_j) and total profits (Π_j) functions are

$$\begin{aligned} ic_i &= \Sigma_t(hc_i hs_{i,t} + bc_i bs_{i,t}) \\ \Pi_i &= \Sigma_{j,t}(r_i - wa_j)o_{ij,t} - ic_i \\ ic_j &= \Sigma_t(hc_j hs_{j,t} + bc_j bs_{j,t}) \\ \Pi_j &= \Sigma_t((wa_j - ca_j)\min(\Sigma_i o_{ij,t}, K_j/n) + (wa_j - ck_j)\max(\Sigma_i o_{ij,t} - K_j/n, 0)) - ic_j \end{aligned} \tag{20}$$

where ca_j is the cost of producing one unit of the product within manufacturer j's capacity. To produce one unit of the product outside the capacity, a higher cost ck_j incurs i.e. due to overtime.

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