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OPTIMIZATION OF ENTERPRISE WORK ON THE BASIS OF ECONOMETRIC MODELING AND SOFTWARE

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Abstract. The article proposes an approach to optimization of basic economic parameters (volume of produced and sold products, cost of production, unit price of production, earned profit) using econometric modeling and Excel and Mathcad software. The developed mathematical model and the method of its computer implementation make it possible to make an informed and effective decision by the management of the enterprise to optimize the enterprise operation in order to obtain maximum profit. To build an econometric model, a sample series of data for 36 periods was compiled, determined from the statistical reporting of 18 enterprises. This allowed us to obtain accurate estimates of the parameters of the dependence of the unit price of production, the cost of all products from its volume. Based on the compiled econometric models, the density of the correlation between the studied indicators was estimated, and the adequacy of the obtained models was proved. The estimated one-factor dependences of the unit price of production, costs on the volume of produced and sold products made it possible on the basis of marginal analysis to identify the optimal volume of production, which, in turn, allowed us to determine the optimal values of the unit price of such products, the cost of its manufacture and the maximum possible level of enterprise profit. This approach allows: optimization of basic performance of the enterprise, which allows to maximize its profits at this stage; forecasting the future values of the dependent feature - unit price or production costs (or independent feature - quantity of produced and sold products) under the condition of a fixed value of the independent (or dependent) feature. In addition, on the basis of the obtained optimal values of basic indicators of the enterprise their deviation from those values that exist at the enterprise for the last period of its work is determined, which allows us to define clearly the directions of their optimization with subsequent application of economic analysis.

Keywords. correlation regression dependence, linear one-factor regression, parabolic regression dependence, correlation, unit price of production, volume of production, production costs, profit, enterprise optimization.

1 Introduction

At the current stage of development of the Ukrainian economy, in the context of the coronavirus pandemic and the global and domestic financial crises caused by it,

the growth of competition in the market of goods and services, the optimal management of the enterprise becomes especially necessary to achieve the maximum possible commercial goal. With the decline in demand for certain types of services, the question of optimizing existing business processes in enterprises by developing appropriate mathematical models using modern software sharply arises.

To analyze, optimize or restructure its activities, the enterprise as an entity needs to implement an optimal model of its business processes, which reflects its basic indicators, as well as financial and other resource components for each process. This model provides visual material for analysis, shows the «bottlenecks» in its activities, identifies possible risks and unproductive costs borne by the company in its activities due to duplication of functions and areas of responsibility or, conversely, «irresponsibility». At the same time, an aggressive market requires caution in decision-making, dictates the need to reduce risks [1]. Under these conditions, the relevance of finding rational ways to manage the enterprise increases significantly, which becomes possible only with the use of optimal methods of econometric modeling of business processes using modern software.

Thus, the purpose of the article is to maximize the enterprise's profit by developing and applying appropriate correlation-regression models of factors influencing the enterprise's profit and methods of its optimization using modern software packages.

The methods used in the study include: correlation-regression approach based on the method of least square deviations, which is used to build one-way relationships of unit price and cost of production and check the correlation of the relationship between the studied features; the method of marginal analysis – to find the optimal value of the enterprise's profit, as well as methods of comparison and substitution used to predict the values of the dependent feature – unit price or cost of production (or independent feature – volume of produced and sold products) sold under a fixed value of independent (or dependent) features; method of economic analysis – to develop recommendations and proposals for approximation of the values of the studied indicators available at the enterprise for the last period to the optimal ones.

Software used to build econometric models are Excel spreadsheet and mathematical software package MathCad.

2 Literature analyzes

We will analyze related works and consider the main models used to optimize certain economic parameters in the works of both domestic and foreign scientists.

Considering the stochastic model of the compromise between costs and profits in the works [2-3], the authors solve the problem of optimizing these parameters with the maximum customer satisfaction, the maximum profit of investors of the facility, and the minimum transportation cost of its oriented-customers. The authors note that in practice, some factors of the Facility Location Allocation (FLA) problem are usually changing and the problem features with uncertainty. To account for this uncertainty, some researchers have addressed the stochastic profit and cost issues of FLA. To handle this issue via a more practical manner, it is essential to address the cost-profit tradeoff issue of FLA. By taking the vehicle inspection station as a typical

automotive service enterprise example, this work presents new stochastic cost-profit tradeoff FLA models with region constraints [2]. A hybrid algorithm integrating stochastic simulation and Genetic Algorithms (GA) is proposed to solve the proposed models. Some numerical examples are given to illustrate the proposed models and the effectiveness of the proposed algorithm. However, these works focus more on the location of the business object and its impact on the costs and profits of the enterprise.

In the work [4], managing costs and cost structure throughout the value chain are considered. The author has developed a theoretical model that links strategic cost management with strategy development and performance evaluation, taking into account divergence and analysis of variance. The disadvantage of this work is its purely theoretical nature without presenting a specific formalized model.

In the work [5] the pricing performed within the local models of stochastic volatility Local Stochastic Volatility (LSV) is considered in detail, algorithms for estimating the model parameters are described in detail and emphasis is placed on presenting practical details regarding the setup and the numerical solution of both forward and backward partial differential equation (PDEs) / partial integro-differential equation (PIDEs) obtained from the LSV models. The specificity of this work is that the presented quantitative methods and algorithms are used mainly in the currency and securities markets.

Questions about revenue optimization to address the operation and maintenance cost of a data center are discussed in detail in the work of Snehanshu S. [6-8]. This paper proposes an algorithmic/analytical approach to address the issues of optimal utilization of resources towards a feasible and profitable model. The economic sustainability of such a model is accomplished via Cobb-Douglas production function. The production model seeks to answer questions on maximal revenue given a set of budgetary constraints. The model suggests minimum investments needed to achieve target output. However, in these works, the optimization models are quite specific and adapted to the features of the data center.

The article [9] presents a multifactor correlation-regression model of the dependence of the unit price and the internal factors of influence on it, which is estimated on the basis of the corresponding coefficients of elasticity, but in this work no method of optimization of parameters is worked out.

The purpose of the article [10] is to solve problems of forecasting the real price of the option using evolutionary and genetic algorithms that affect the accuracy of price forecasting. To achieve this goal, genetic and evolutionary algorithms are used in the fields of financial instruments, to create software that is designed to analyze and forecast the real price option.

In the work [11] it is noted that the production process of manufacturing enterprises is usually limited by production techniques and equipment. The quantity of production is also restricted by equipment capacity and security production, therefore in order to lower cost, process enterprises have to carry out profit optimization according to the product price and product sales. Considering the demand of process manufacturing enterprise, was optimize the cost of working procedure and dealing with the intermediate and castoff and put forward a new type process manufacturing optimization model.

In the work [12] a model for profit optimization and management is presented. It takes into account both the quantity of sales, prices, costs and other factors, as well as

new factors related to market and competitors – market share, prices, quality and marketing costs of competitors and others. There are listed features, limitations and advantages of the model. For more clarity, the presentation of the model is accompanied by two main types of tasks related to optimal prices, strategies and costs for the industrial enterprise.

Also, some aspects of modeling and optimization of sample parameters at both the macro and micro levels were considered in works [13-20], but these works did not take into account all aspects that are taken into account in the mathematical model proposed by the authors of this article, and no correlations between the analyzed parameters were determined.

3 Formal problem statement

During the optimization processes at the enterprise there is a need to take into account a set of statistics on the volume of produced and sold products, unit prices, expenditures at full cost of production and gross profit of the enterprise for at least 36 periods, provided there is a close correlation between them, as well as the need to achieve the condition of profit maximization. The authors propose to develop correlation-regression one-factor dependences of unit price, as well as costs on the volume of produced and sold products based on software packages Excel and mathematical package MathCad. This will allow using the methods described above to identify the optimal value of the enterprise's profit and develop proposals for its maximization with the subsequent use of methods of economic analysis.

We will build a method of maximizing the enterprise's profit on the basis of one-factor correlation and functional analysis using the following multi-stage approach.

We introduce the following notation:

V is the volume of produced and sold products (or services);

P_v is unit price, $v = 1 \dots V$;

$P_v \cdot V$ is revenue from sales of goods (services);

C is the cost of volume V of products;

R is profit from sales of V products.

Stage 1. The choice of the form of connection between the studied indicators: the price P_v per unit of production and the volume V of produced and sold products, as well as between the costs C for the entire volume of production and the volume V . This process becomes possible by plotting relevant empirical data and comparing increments over different periods with increments corresponding to known linear or nonlinear dependencies.

Stage 2. For the selected forms of dependences, it is necessary to construct corresponding one-factor regression models, estimating values of their parameters. To do this, special correlation tables are made, which determine the amount needed to estimate the parameters. The obtained parameter estimates allow us to write the required regression equations.

Step 3. It is necessary to evaluate the density of the relationship between performance and factor characteristics using the correlation coefficient and check the adequacy of the constructed models using the coefficient of determination.

Step 4. The obtained correlation equations should be used during the marginal analysis to obtain the optimal value of output, based on which it is necessary to estimate the optimal values for the unit price, costs for the entire output and the maximum possible profit of the enterprise.

To implement this process, we note that the volume of production, unit price and costs of its manufacture and sales are in certain dependencies, so obtaining maximum profit from sales is possible only with specific ratios between these values. Therefore, obtaining the maximum profit from sales can be described by the following mathematical model:

$$R_{(V)} = P_v \cdot V - C, \quad (1)$$

A necessary condition for the extremum of this function is a derivative of V , which should be equal to 0:

$$R_{(V)}' = (P_v \cdot V)' - C' = 0, \quad (2)$$

where $R(V)'$ is marginal profit from the sale of volume V of products;

$(P_v \cdot V)'$ is marginal revenue;

C' is marginal cost of volume V products.

Based on this equation we have:

$$(P_v \cdot V)' = C', \quad (3)$$

This ratio allows us to analyze the optimal volume of produced and sold products (services) by the criterion of maximum profit from sales.

The condition of the maximum is that the derivative $R_{(V)}'$ at the point of maximum is equal to 0: $R_{(V_{opt})}' = 0$, moreover $R'_{(V_{opt} - 1)} > 0$, and at the point $(V_{opt} + 1)$ derivative $R'_{(V_{opt} + 1)} < 0$.

4 Experimental Model

The peculiarity of the decision-making process for the optimization of certain indicators is to take into account the density of correlations between certain interdependent factors, as well as the production capacity of the enterprise. In the proposed approach, the authors use the above indicators of 18 enterprises for 36 periods. Table 1 provides the initial data for Enterprise 1.

Table 1. Initial data for Enterprise 1

<i>Period, i</i>	V_i	P_{V_i}	C_i	<i>Period, i</i>	V_i	P_{V_i}	C_i
1	225	48	7875	19	135	105	12048,75
2	220	50	8250	20	130	105	11602,5
3	215	55	8868,75	21	125	110	11687,5
4	210	55	9240	22	120	115	11730
5	205	55	9020	23	115	120	11730

6	200	60	9600	24	110	125	11687,5
7	195	65	10140	25	105	135	11340
8	190	65	9880	26	100	140	11200
9	185	70	10360	27	95	150	11400
10	180	70	10458	28	90	150	10800
11	175	80	11200	29	85	160	10880
12	170	85	11560	30	80	165	10560
13	165	90	11880	31	75	170	10200
14	160	90	11520	32	70	170	9520
15	155	95	11780	33	65	175	9100
16	150	95	12112,5	34	60	180	8640
17	145	98	12078,5	35	55	190	8360
18	140	100	11900	36	50	200	8000

Stage 1.1. To choose the form of connection $\hat{P}_v = P(V)$ between the price P_v per unit of output and the volume V of produced and sold products, we build a graph of relevant empirical data (using Excel spreadsheet) and approximate them with one of the known dependencies.

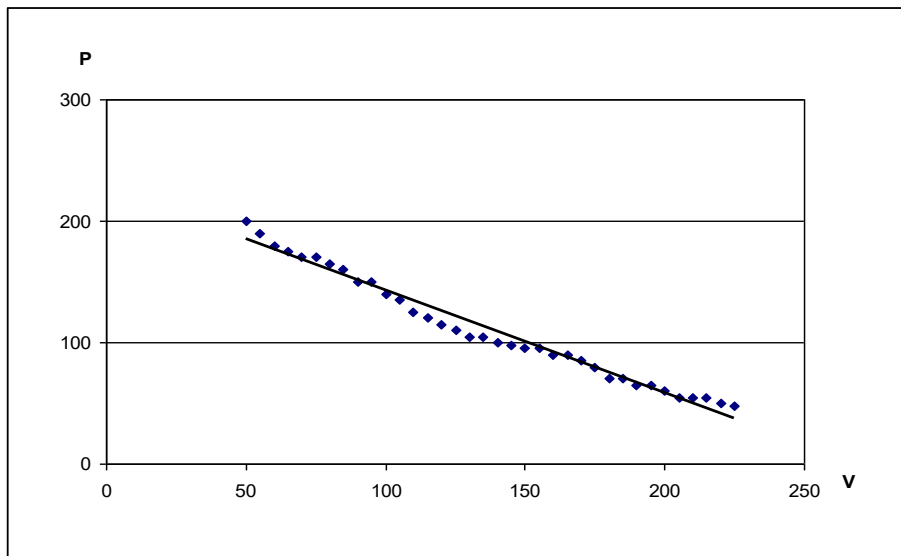


Fig. 1. Approximation of empirical data for $\hat{P}_v = P(V)$ by linear dependence

Based on the graph shown in Fig. 1, the dependence $\hat{P}_v = P(V)$ must be approximated linearly.

Stage 2.1. The form of linear one-factor regression dependence of the unit price on the volume of production is as follows [21]:

$$\hat{P}_v = b_0 + b_1 \cdot V_i. \quad (4)$$

Estimates of the parameters b_1 and b_0 in the following linear one-factor model are obtained using dependences (5) and (6):

$$b_1 = \frac{\sum_{i=1}^n V_i P_{v_i} - \frac{\sum_{i=1}^n V_i \cdot \sum_{i=1}^n P_{v_i}}{n}}{\sum_{i=1}^n V_i^2 - \frac{1}{n} \left(\sum_{i=1}^n V_i \right)^2}, \quad (5)$$

$$b_0 = \bar{P} - b_1 \bar{V}. \quad (6)$$

To calculate the data specified in dependencies (5) and (6), we use the spreadsheet Excel and make the appropriate correlation Table 2.

Therefore, we calculate on the basis of dependences (5) and (6) of the parameter estimation for Enterprise 1:

$$b_1 = \frac{467285 - \frac{4950 \cdot 3993}{36}}{777750 - \frac{1}{36} (4950)^2} = -0,8435; \quad b_0 = 110,917 + 0,84179 \cdot 137,5 = 226,85.$$

Thus, the obtained correlation regression dependence based on the calculated parameter estimates for Enterprise 1 takes the form:

$$\hat{P}_{v_i} = 226,85 - 0,8435 \cdot V_i. \quad (7)$$

Stage 3.1. Based on the spreadsheet Excel correlation Table 2, the corresponding sums of squares for SSR and SST (in columns 7 and 8 of table 2) were obtained to estimate the correlation and determination coefficients based on the dependence:

$$R = \pm \sqrt{D} = \pm \sqrt{\frac{SSR}{SST}} = \pm \sqrt{\frac{\sum_{i=1}^n (\hat{P}_{v_i} - \bar{P})^2}{\sum_{i=1}^n (P_i - \bar{P})^2}}, \quad (8)$$

Since when the unit price increases, the number of sold products steadily decreases, the correlation between them is negative, so choose the sign «-» before the root:

Table 2. Calculation of parameters of linear one-factor dependence

	A	B	C	D	E	F	G	H
1	N_i	V_i	P_i	V_i^2	$V_i P_i$	\hat{P}_i	$(\hat{P}_i - \bar{P})^2$	$(P_i - \bar{P})^2$
2	1	225	48	10800	50625	37,05255	3951,51929	5447,7033
3	2	220	50	11000	48400	41,27018	3704,07485	4842,8971
4	3	215	55	11825	46225	45,48782	3120,46373	4273,6677
5	4	210	55	11550	44100	49,70545	3120,46373	3740,0151
6	5	205	55	11275	42025	53,92308	3120,46373	3241,9394
7	6	200	60	12000	40000	58,14071	2586,85262	2779,4405
8	7	195	65	12675	38025	62,35834	2103,24151	2352,5184
9	8	190	65	12350	36100	66,57598	2103,24151	1961,1732
10	9	185	70	12950	34225	70,79361	1669,6304	1605,4048
11	10	180	70	12600	32400	75,01124	1669,6304	1285,2133
12	11	175	80	14000	30625	79,22887	952,408179	1000,5986
13	12	170	85	14450	28900	83,4465	668,797068	751,5607
14	13	165	90	14850	27225	87,66414	435,185957	538,09967
15	14	160	90	14400	25600	91,88177	435,185957	360,21548
16	15	155	95	14725	24025	96,0994	251,574846	217,90813
17	16	150	95	14250	22500	100,317	251,574846	111,17762
18	17	145	98	14210	21025	104,5347	165,408179	40,023943
19	18	140	100	14000	19600	108,7523	117,963735	4,4471047
20	19	135	105	14175	18225	112,9699	34,3526235	4,4471047
21	20	130	105	13650	16900	117,1876	34,3526235	40,023943
22	21	125	110	13750	15625	121,4052	0,74151235	111,17762
23	22	120	115	13800	14400	125,6228	17,1304012	217,90813
24	23	115	120	13800	13225	129,8405	83,5192901	360,21548
25	24	110	125	13750	12100	134,0581	199,908179	538,09967
26	25	105	135	14175	11025	138,2757	582,685957	751,5607
27	26	100	140	14000	10000	142,4934	849,074846	1000,5986
28	27	95	150	14250	9025	146,711	1531,85262	1285,2133
29	28	90	150	13500	8100	150,9286	1531,85262	1605,4048
30	29	85	160	13600	7225	155,1462	2414,6304	1961,1732
31	30	80	165	13200	6400	159,3639	2931,01929	2352,5184
32	31	75	170	12750	5625	163,5815	3497,40818	2779,4405
33	32	70	170	11900	4900	167,7991	3497,40818	3241,9394
34	33	65	175	11375	4225	172,0168	4113,79707	3740,0151
35	34	60	180	10800	3600	176,2344	4780,18596	4273,6677
36	35	55	190	10450	3025	180,452	6262,96373	4842,8971
37	36	50	200	10000	2500	184,6697	7945,74151	5447,7033
38	Σ	4950	3991	466835	777750	3991	70736,3056	69108,008
39	Σ/n	137,5	110,8611	12967,64	21604,17	110,8611	SST	SSR

$$R = -\sqrt{D} = -\sqrt{\frac{SSR}{SST}} = -\sqrt{\frac{\sum_{i=1}^n (\hat{P}_{V_i} - \bar{P})^2}{\sum_{i=1}^n (P_i - \bar{P})^2}}, \quad (9)$$

$$R = \sqrt{\frac{68813,089}{70488,75}} = \sqrt{0,97623} = 0,98804.$$

Since the correlation coefficient $R = 0,988$, the relationship between the unit price and the volume of units produced is tight.

The value of the coefficient of determination $D = 0,976$ indicates the adequacy of the constructed regression dependence $\hat{P}_v = P(V)$.

Stage 1.2. We determine the type of dependence $C(V)$ using a graph of points with coordinates (V_i, C_i) by means of the spreadsheet Excel, as shown in Fig. 2.

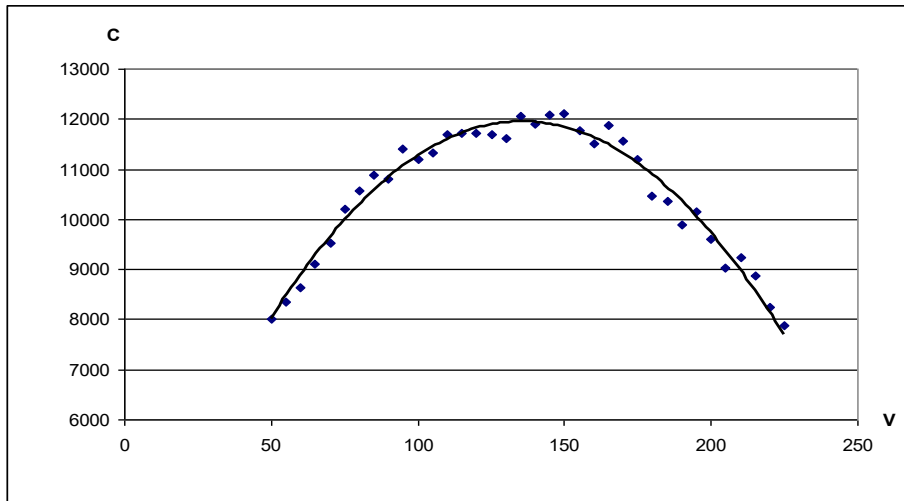


Fig. 2. Approximation of empirical data for by parabolic dependence

Based on the graph of Fig. 2, it is better to approximate the parabolic dependence, the form of which is as follows:

$$\hat{C}_i = b_0 + b_1 \cdot V_i + b_2 \cdot V_i^2. \quad (10)$$

Stage 2.2. To search for unknown parameters b_0, b_1, b_2 of such parabolic dependence, we use the appropriate system of normal equations:

Calculated in Table 3 sums must be substituted for the above-described system of normal equations (11), then we obtain:

$$\begin{cases} 36 \cdot b_0 + 4950 \cdot b_1 + 777750 \cdot b_2 = 378212 \\ 4950 \cdot b_0 + 777750 \cdot b_1 + 133650000 \cdot b_2 = 51820290 \\ 777750 \cdot b_0 + 133650000 \cdot b_1 + 24356861250 \cdot b_2 = 8008548350 \end{cases} \quad (12)$$

We use the Mathcad software package to solve a complex system of linear equations and calculate the parameters b_0 , b_1 , b_2 . The results of Mathcad application are presented in Fig. 3.

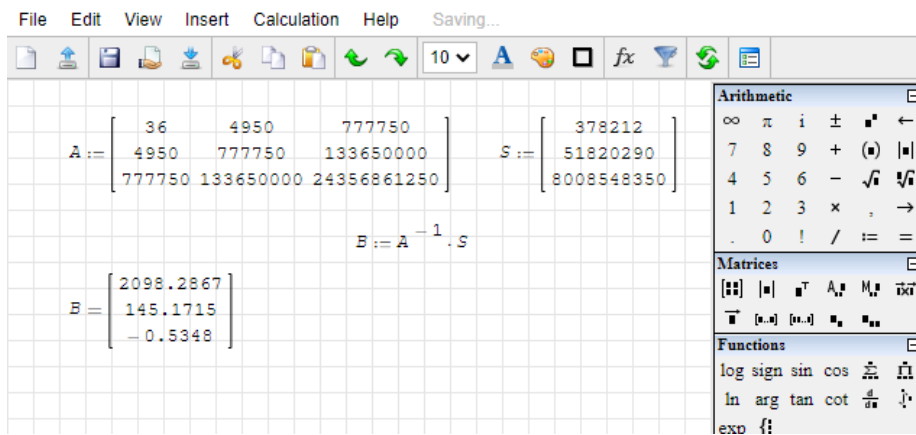


Fig. 3. Calculation of estimates of parameters b_0 , b_1 , b_2 in Mathcad

Therefore, $b_0 = 2098,2867$; $b_1 = 145,1715$; $b_2 = -0,5348$. Thus, equation (10) takes the form:

$$\hat{C}_i = 2098,2867 + 145,1715 \cdot V_i - 0,5348 \cdot V_i^2. \quad (13)$$

Stage 3.2. On the basis of the correlation Table 3 compiled by means of the Excel spreadsheet, the corresponding sums of squares for SSR and SST (in 11 and 10 columns of Table 3) for estimation of correlation and determination coefficients on the basis of dependence are received:

$$R = \sqrt{D} = \sqrt{\frac{SSR}{SST}} = \sqrt{\frac{\sum_{i=1}^n (\hat{C}_i - \bar{C})^2}{\sum_{i=1}^n (C_i - \bar{C})^2}}. \quad (14)$$

We substitute the obtained sums of squares for SSR and SST to this expression and get:

$$R = \sqrt{\frac{60160649,20}{61994909,6}} = \sqrt{0,9704} = 0,9851$$

Since the correlation coefficient $R = 0,9851$, the relationship between expenditures at full cost and the volume of units produced is very close.

The value of the coefficient of determination $D = 0,97$ indicates the adequacy of the constructed regression dependence $\hat{C} = C(V)$.

For the other 17 enterprises, the corresponding regression dependences were also obtained by a similar approach and their adequacy was checked, as shown in Table 4.

Table 4. Results of assessment and verification of adequacy of dependencies

$\hat{P} = P(V)$ and $\hat{C} = C(V)$ for 18 enterprises

№	$\hat{P} = P(V)$	D	R	$\hat{C} = C(V)$	D	R
1	$\hat{P} = -0,8435V + 226,85$	0,976	0,988	$\hat{C} = -0,5348V^2 + 145,17V + 2098,3$	0,97	0,985
2	$\hat{P} = -6,0103V + 135,39$	0,973	0,986	$\hat{C} = -1,6511V^2 + 27,285V + 475,46$	0,832	0,912
3	$\hat{P} = -2,5089V + 97,206$	0,851	0,903	$\hat{C} = -3,7013V^2 + 113,25V - 125,65$	0,898	0,948
4	$\hat{P} = -0,0133V + 24,725$	0,898	0,948	$\hat{C} = -0,0047V^2 + 7,6382V + 5061,5$	0,924	0,961
5	$\hat{P} = -0,0391V + 10,791$	0,933	0,966	$\hat{C} = -0,0488V^2 + 12,467V - 134,45$	0,934	0,966
6	$\hat{P} = -0,0201V + 10,911$	0,879	0,937	$\hat{C} = -0,0122V^2 + 6,0297V - 102,51$	0,917	0,957
7	$\hat{P} = -0,0195V + 41,971$	0,93	0,96	$\hat{C} = -0,0025V^2 + 4,5578V + 6234,1$	0,822	0,907
8	$\hat{P} = -2,7384V + 132,12$	0,959	0,979	$\hat{C} = -0,3217V^2 + 10,089V + 508,29$	0,837	0,915
9	$\hat{P} = -0,2425V + 14,013$	0,973	0,986	$\hat{C} = -0,1224V^2 + 6,6379V + 61,94$	0,945	0,972
10	$\hat{P} = -0,589V + 30,698$	0,811	0,901	$\hat{C} = -0,2451V^2 + 12,686V + 150,5$	0,885	0,941
11	$\hat{P} = -0,3438V + 93,116$	0,984	0,991	$\hat{C} = -0,3093V^2 + 82,954V + 647,32$	0,909	0,953
12	$\hat{P} = -0,0185V + 12,808$	0,927	0,963	$\hat{C} = -0,0125V^2 + 6,8102V + 611,08$	0,885	0,941
13	$\hat{P} = -0,003V + 16,32$	0,966	0,979	$\hat{C} = -0,0037V^2 + 19,547V - 4301,6$	0,856	0,925
14	$\hat{P} = -0,0041V + 11,298$	0,892	0,944	$\hat{C} = -0,0051V^2 + 12,961V - 1533,4$	0,973	0,986
15	$\hat{P} = -0,2924V + 103,04$	0,830	0,911	$\hat{C} = -0,5948V^2 + 159,98V - 3337,7$	0,754	0,869
16	$\hat{P} = -0,0058V + 21,144$	0,94	0,970	$\hat{C} = -0,0034V^2 + 11,449V$	0,979	0,989

				- 1587,7		
17	$\hat{P} = -0,0139V + 36,966$	0,967	0,983	$\hat{C} = -0,004V^2 + 12,01V - 535,6$	0,937	0,967
18	$\hat{P} = -0,3486 V + 114,83$	0,982	0,99	$\hat{C} = -0,2348V^2 + 79,229V + 761,57$	0,978	0,988

5 Implementation of the model

We apply the obtained estimated equations to $\hat{P}_v(V)$ and $\hat{C}(V)$ to determine the optimal profit of the studied enterprises.

Step 4. We evaluate the profit \hat{R}_V by the following relationship:

$$\hat{R}_V = \hat{P}_v(V) \cdot V - \hat{C}(V). \quad (15)$$

Substituting in this dependence instead of $\hat{P}_v(V)$ the expression for the estimated price (4), and instead of $\hat{C}(V)$ the expression for the estimated costs (10) we obtain the equation for finding the estimated profit of the enterprise.

$$\begin{aligned} \hat{R}_V = & (-0,8435V + 226,85) \cdot V - (-0,5348V^2 + 145,17V + 2098,3) = -0,8435V^2 + 226,85V + \\ & + 0,5348V^2 - 145,17V - 2098,3 = -0,3087V^2 + 81,68V - 2098,3. \end{aligned}$$

The obtained equation describes the parabolic dependence of profit on the volume of output. To find the maximum value of profit in such a dependence, it is necessary to find its extremum. The extremum of this function is a derivative of V , which should be equal to 0, according to dependence (2):

$$\begin{aligned} R(V)' &= 81,68 - (0,3087 \cdot 2) \cdot V = 0 \\ 81,68 - 0,6174 \cdot V &= 0 \\ V &= \frac{-81,68}{-0,6174} = 132,3. \end{aligned}$$

Let us check whether the obtained extremum of the function ($V = 132,3$) is its maximum. The condition of the maximum is: $R'_{(V_{opt})} = 0$, and $R'_{(V_{opt}-1)} > 0$, $R'_{(V_{opt}+1)} < 0$:

$$R'_{(v=132)} = 0,1832 > 0, \quad R'_{(v=133)} = -0,4342 < 0,$$

Therefore, $V=132,3 = V_{opt}$ is the point of optimum, where the criterion of optimality is the maximum profit from production.

Based on the obtained optimal value of the volume of output $V_{opt} = 132,3$ we determine the optimal cost \hat{C}_{opt} and optimal price \hat{P}_{opt} based on expressions (7) and (13), respectively:

$$\hat{P}_{opt} = -0,8435 \cdot 132,3 + 226,85 = 115,255$$

$$\hat{C}_{opt} = -0,5348 \cdot 132,3^2 + 145,17 \cdot 132,3 + 2098,3 = -0,5348 \cdot 17503,29 + 19205,991 + 2098,3 = -9360,759 + 21304,291 = 11943,532.$$

Based on the obtained optimal values for the volume of products V_{opt} , the optimal unit price $\hat{P}_{V_{opt}}$ and the estimated optimal costs \hat{C}_{opt} at full cost of production, we calculate the maximum profit that the enterprise should receive under the condition of rational management in the following dependence:

$$\hat{R}_{opt} = \hat{P}_{V_{opt}} \cdot V_{opt} - \hat{C}_{opt}. \quad (17)$$

$$\hat{R}_{opt} = 115,255 \cdot 132,3 - 11943,532 = 3304,698.$$

Using the author's approach, the optimal indicators for 18 analyzed enterprises were determined, which are listed in Table 5.

Table 5. The estimated optimal indicators of 18 enterprises

№	$\hat{R}_V = \hat{P}_V(V) \cdot V - \hat{C}(V)$	V_{opt}	$\hat{P}_{V_{opt}}$	\hat{C}_{opt}	\hat{R}_{opt}
1	$\hat{R}_V = -0,3087V^2 + 81,68V - 2098,3$	132	115,055	11943,53	3304,698
2	$\hat{R}_V = -4,3595V^2 + 108,1050V - 75,46$	12	63,266	565,12	194,075
3	$\hat{R}_V = 1,1924V^2 - 16,044V + 125,65$	7	79,644	485,74	71,770
4	$\hat{R}_V = -0,0086V^2 + 17,0868V - 5061,5$	993	11,518	8011,80	3425,671
5	$\hat{R}_V = 0,0097V^2 - 1,6760V + 134,45$	86	7,428	576,79	62,055
6	$\hat{R}_V = -0,0079V^2 + 4,8813V + 102,510$	309	4,700	595,80	856,532
7	$\hat{R}_V = -0,017V^2 + 37,4132V - 6234,1$	1100	20,521	8222,68	14350,42
8	$\hat{R}_V = -2,4167V^2 + 122,031V - 08,29$	25	62,982	557,95	1032,195
9	$\hat{R}_V = -0,1201V^2 + 7,3751V - 61,94$	31	6,496	150,09	51,272
10	$\hat{R}_V = -0,3439V^2 + 18,012V - 150,5$	26	15,384	314,65	85,336
11	$\hat{R}_V = -0,0345V^2 + 10,162V - 47,32$	147	42,577	6157,89	100,984
12	$\hat{R}_V = -0,006V^2 + 5,9978V - 611,08$	500	3,558	891,18	887,820
13	$\hat{R}_V = 0,0007V^2 - 3,227V + 4301,6$	2305	9,405	21096,04	582,483
14	$\hat{R}_V = 0,001V^2 - 1,663V + 1533,4$	832	7,887	5719,81	842,008
15	$\hat{R}_V = 0,3024V^2 - 56,94V + 3337,7$	94	75,554	6444,77	657,346
16	$\hat{R}_V = -0,0024V^2 + 9,695V + 1587,7$	2020	9,428	7665,92	11378,64
17	$\hat{R}_V = -0,0099V^2 + 24,956V + 535,6$	1260	19,452	8246,60	16262,92
18	$\hat{R}_V = -0,1138V^2 + 35,601V - 61,57$	156	60,302	7409,66	2022,769

We compare the results of the enterprise performance for the last 36th period with the optimal values estimated by the approach proposed by the authors of the article. All calculations are summarized in Table 6.

Table 6. Comparative characteristics of the enterprise indicators for the last period with the proposed optimal values

Parameters	V_i	P_{vi}	C_i	R_v
Firm 1				
36 period	50	200	8000	2000
Optimal	132	115,055	11943,532	3304,698
Deviation	-82	84,945	-3943,532	-1304,6984
Firm 2				
36 period	5	108	529	11
optimal	12	63,266	565,12	194,075
deviation	-7	44,7336	-36,1216	-183,0752
Firm 3				
36 period	5	82	377	33
optimal	7	79,644	485,74	71,770
deviation	-2	2,3563	-108,7363	-38,7696
Firm 4				
36 period	300	23	6555	345
optimal	993	11,518	8011,80	3425,671
deviation	-693	11,4819	-1456,8023	-3080,671
Firm 5				
36 period	44	8,2	308	52,8
optimal	86	7,428	576,79	62,055
deviation	-42	0,7716	-268,7872	-9,2552
Firm 6				
36 period	88	8,2	308	413,6
optimal	309	4,700	595,80	856,532
deviation	-221	3,4999	-287,7991	-442,9318
Firm 7				
36 period	150	45	6555	195
optimal	1100	20,521	8222,68	14350,420
deviation	-950	24,479	-1667,68	-14155,42
Firm 8				
36 period	10	108	529	551
optimal	25	62,982	557,95	1032,195
deviation	-15	45,017603	-28,949123	-481,19547
Firm 9				

36 period	18	10	140	40
optimal	31	6,496	150,09	51,272
deviation	-13	3,5045	-10,0885	-11,272
Firm 10				
36 period	10	30	250	50
optimal	26	15,384	314,65	85,336
deviation	-16	14,616	-64,6484	-35,3356
Firm 11				
36 period	40	80,873433	3234,9373	35,318766
optimal	147	42,577	6157,89	100,984
deviation	-107	38,296033	-2922,957	-65,664734
Firm 12				
36 period	180	10	1500	300
optimal	500	3,558	891,18	887,820
deviation	-320	6,442	608,82	-587,82
Firm 13				
36 period	1350	11,9	16020	45
optimal	2305	9,405	21096,04	582,483
deviation	-955	2,495	-5076,0425	-537,4825
Firm 14				
36 period	440	8,2	3080	528
optimal	832	7,887	5719,81	842,008
deviation	-392	0,3132	-2639,8096	-314,008
Firm 15				
36 period	60	85	4770	330
optimal	94	75,554	6444,77	657,346
deviation	-34	9,4456	-1674,7672	-327,3464
Firm 16				
36 period	1000	16	6555	9445
optimal	2020	9,428	7665,92	11378,640
deviation	-1020	6,572	-1110,92	-1933,64
Firm 17				
36 period	1000	23	7555	15445
optimal	1260	19,452	8246,60	16262,920
deviation	-260	3,548	-691,600	-817,92
Firm 18				
36 period	50	100	4000	1000
optimal	156	60,302	7409,66	2022,769
deviation	-106	39,697718	-3409,6629	-1022,7692

Let us analyze the data on Enterprise 1: comparing the performance of the enterprise for the 36th period with the estimated by the authors of the optimal values of output, unit price and expenditures at full cost, note that: in order to increase profits by 1278,245 monetary units, it is necessary to increase the volume of production by 82 units and due to this it is possible to reduce the unit price by 84,945 monetary units, and the costs should increase by 3943,532 monetary units.

Ways to bring the values available for the 36th period to the level of optimal at the studied enterprises can be worked out using the methods of economic analysis.

6 Conclusions

The linear $\hat{P} = P(V)$ and parabolic $\hat{C} = C(V)$ dependences are fairly accurate approximations for the initial data on P , V and C as evidenced by the obtained values of the coefficients of determination.

The approach proposed by the authors of the article allows optimal management of the enterprise, helping the head of the enterprise to make an informed and effective decision to obtain the maximum possible profit, based on the available capabilities of the enterprise.

The automation of the proposed estimated mathematical models was carried out using the Excel spreadsheet and a mathematical software package Mathcad.

The practical value of the obtained results is that compiled with the use of correlation-regression and optimization methods, as well as software one-factor dependences of unit price, as well as costs of output and sold products allowed us (using methods of economic and marginal analysis) to identify optimal value of the volume of output, on the basis of which the optimal values for the unit price of production, the cost of its manufacture and the maximum possible level of enterprise's profit were determined. This approach allows not only optimizing the basic performance of the enterprise, which allows maximizing its profits at this stage, but it also allows predicting the future values of the dependent feature – unit price or production costs (or independent feature – volume of output) provided that the value of the independent (or dependent) feature is fixed.

In addition, on the basis of the obtained optimal values of basic indicators of the enterprise we determined their deviation from those values that exist at the enterprise for the last period of its work, which allows us to clearly define the directions of their optimization with subsequent application of economic analysis methods.

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