Robust Optimization of Train Timetable with Short-Turning Strategy Considering Uncertain Passenger Demand and Vehicle Selection

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Abstract

Considering the uncertainty of passenger demand and vehicle selection, this paper investigates a robust optimization approach for the train timetabling problem with short-turning strategy in urban rail transit system. With the scenario-based representation of passenger distribution, a mixed-integer linear programming (MILP) model is formulated that simultaneously integrates train timetabling, short-turning strategy and rolling stock circulation. The proportion of passengers who take the short-turning train services to the last station of the short-turning region and transfer to the full-length train services to their destination stations, is introduced to describe the passenger vehicle selection behavior under short-turning strategy. Finally, three experiments are designed for Xi'an Metro Line 3 to verify the solution quality and effectiveness of the proposed methods. The results indicate that the robust train timetable can more effectively satisfy multi-scenario passenger demand than the satisfactory train timetable generated by independent optimization of each demand scenario.

Keywords

train timetable, robust optimization, short-turning strategy, uncertain passenger demand, vehicle selection

1 Introduction

As one of the most important modes of public transportation, urban rail transit has attracted increasing attention and has increased the travel demand for this mode, owing to the outstanding advantages of faster velocity, higher reliability, and larger capacity. The number of passengers commuting with urban rail transit systems is more than 10 million per day in big cities, such as Beijing, Shanghai, and Tokyo. To maximize the transportation capacity and improve passenger service quality, the train timetable optimization should consider the coupling relationship between passengers and train services under the passenger distribution.

Passenger demand is characterized by temporal and spatial distributions. At some stations, there is a high arrival rate for passenger demand during peak hours, causing some stranded passengers to remain at the stations. Meanwhile, fewer passengers at other stations of the same transit line, resulting in low occupancy levels for transportation capacity. Some researches indicate train timetabling with short-turning strategy (Li et al., 2019; Yang et al., 2021; Zhu et al., 2022) can better adapt to the unbalanced temporal and spatial passenger demand distributions with smaller fleet sizes. Short-turning strategy means that some train services (full-length) pass through all stations along this line, while some train services (short-turning) only travel through several stations.

Furthermore, passengers usually need not refer to the train timetable when they take trains (Gong et al., 2021), and traffic demand always exhibits a significant degree of uncertainty in daily operations, such as its spatiotemporal distribution diversity and complex passenger vehicle selection, which increases the complexity of train timetable optimization. Therefore, this paper proposes a robust train timetabling approach with short-turning strategy in terms of uncertain passenger demand and vehicle selection.

1.1 Literature review

The train timetable optimization problem has become popular research in recent decades. The train timetable defines the arrival and departure times of all the train services at each station, subject to operational and safety requirements. Due to the uneven characteristics of passenger spatiotemporal distribution, the short-turning strategy has been investigated by some researchers in the transportation systems. Tirachini et al. (2011) developed a short-turning model to increase the service frequency on the more loaded sections for a single bus line-single period setting, considering both operators' and users' costs. Canca et al. (2016) proposed a tactical approach with a short-turning strategy, to increase the frequency among certain stations of the lines and to equilibrate the train occupancy level. Yuan et al. (2021) built a new integrated optimization model for the train timetable, rolling stock assignment, and short-turning strategy on a bidirectional metro line, to reduce passengers' total waiting time on platforms. Yang et al. (2021) built a passenger demand-oriented train scheduling optimization model with a two-layer space-time network considering a flexible short-turning strategy to reduce the total passenger travel time.

Meanwhile, some researchers mainly focus on the integrated optimization of train timetable and rolling stock circulation. Wang et al. (2017) proposed an integrated model to optimize the train schedule and circulation plan simultaneously, which involved the operation of train services, the turnaround operations, the entering/exiting depot operation, and the number of available trains. Hoogervorst et al. (2021) present a Variable Neighborhood Search heuristic for the rolling stock rescheduling problem, which means rescheduling rolling stock when a disruption leads to cancellations in the timetable. Wang et al. (2022) formulated the studied problem as a new binary linear model, to minimize the weighted sum of total deadhead distance and total deadhead running time of rolling stocks during the depot exiting and entering operations, and developed a row and column generation-based algorithm to solve it.

Furthermore, in urban rail transit systems, there is uncertainty regarding passenger demand, some researchers applied robust optimization methods to optimize the train timetable. Shafia et al. (2012) proposed a new robust train-timetabling model and developed a branch-and-bound (B&B) algorithm to minimize the arrival times of trains for a single-track railway line. Jamili and Pourseyed Aghaee (2015) optimized urban rail stop-skipping patterns based on uncertain passenger demand to increase the commercial speed and save energy consumption. Qi et al. (2018) proposed an Integer Linear Programming (ILP) model for integrated train timetabling and stop planning problem considering different demand scenarios. Zhou et al. (2020) built a two-phase robust optimization model considering uncertain passenger arrival times and alighting passengers to minimize total waiting passengers. Gong et al. (2021) formulated an integer nonlinear programming (INLP) model to simultaneously optimize the train services number, headway settings and speed profile with the scenario-based passenger distribution. Zhu et al. (2022) proposed a robust train timetabling optimization model with a short-turning strategy considering the uncertainty in passenger demand, to balance train utilization and stranded passengers.

1.2 Focus of this study

Table 1 summaries the difference between the proposed method and relevant existing approaches. As stated above, few studies have investigated train timetable optimization by considering the passenger uncertainty as well as short-turning strategy and rolling stock circulation simultaneously in the urban rail transit system. In real-world operations, passenger uncertainty includes the following two aspects: passenger demand volume and vehicle selection behavior.

On the one hand, passenger demand volume consistently exhibits an uncertain fluctuation characteristic, and varies greatly on different days or periods. For illustration convenience, Fig. 1 demonstrates the passenger demand curves of five workdays in one week. The passenger demand of one day is typically different from that on other days, mainly larger during the peak hours on Monday and Friday. Recently, some studies (Cacchiani et al., 2020; Gong et al., 2021; Huang et al., 2021) focused on the robust optimization of train timetabling and usually used the scenario-based method to represent the uncertainty of passenger demand. The scenario-based method refers to several scenarios of passenger demand, considered synchronously in the optimization. For instance, the passenger demand of one day in Fig.1 can be considered a scenario.

On the other hand, vehicle selection behavior is not certain, especially in the short-turning strategy, some passengers can't arrive at their destination stations directly by short-turning train services. In other words, they must take the train of full-length services. In most existing research about short-turning strategy, there are assumptions that passengers should wait on the platform and choose the direct trains to reach their destinations (Li et al., 2019; Zhu et al., 2022). In reality, part of these passengers will take a train of short-turning services to the one station of the short-turning region and wait for a train of full-length services to the destination station. This unique "transfer" behavior will change the coupling relationship between passengers and train services and affect train loading factors. Some studies have considered passenger travel behaviors in the train timetable optimization problem. These studies have focused primarily on route choice behavior (Zhu et al., 2014; R. Xu et al., 2021), transfer behavior (X. Xu et al., 2021), train-booking behavior (Zhou et al., 2019), traffic mode choice behavior (Xu et al., 2018), and other travel behaviors under a disruption situation (Veelenturf et al., 2017; Zhu and Goverde, 2019). No attention has been given to extending the vehicle selection behavior to the train timetable optimization with the short-turning strategy.

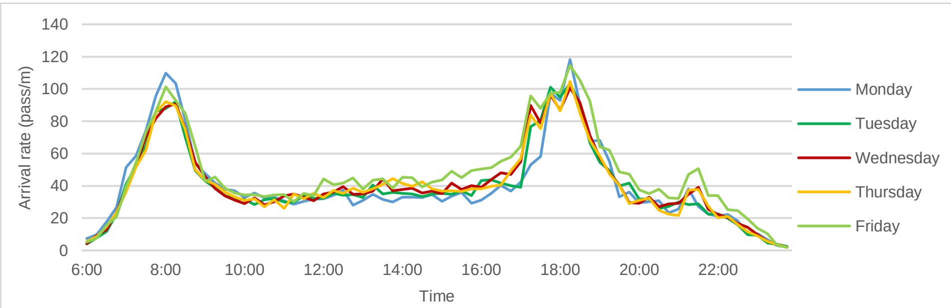


Fig.1 Illustration of passenger demand curves in one week

Table 1 Comparison between this paper and existing literature

Publication	Approach	Passenger Demand Representation	Objective	Solution methods
(Jamili and Aghaee, 2015)	TT-SS-SL	Dynamic; TDOD matrix	PTT	DB; SA
(Qi et al., 2018)	TT-SS-SL	Dynamic; Scenario-based TDOD matrix	PS	CPLEX
(Wang et al., 2018)	TT-RS-DL	Static; Sectional passenger	TLV, HD, RSN	CPLEX
(Zhang et al., 2018)	TT-RS-ST-DL	Null	HD, RSN	Solver
(Li et al., 2019)	TT-ST-SL	Static; TDOD matrix	TLV, OC	GA
(Zhou et al., 2020)	TT-SL	Stochastic; Scenario-based arrival rate	PWN	GA
(Yuan et al., 2021)	TT-RS-ST-DL	Dynamic; TDOD-matrix	PWT	GA; Solver
(Yang et al., 2021)	TT-RS-ST-DL	Dynamic; TDOD-matrix	PTT	Lagrangian
(Gong et al., 2021)	TT-SL	Stochastic; Scenario-based TDOD matrix	FC, PWT, EC	CPLEX, VNS
(Zhu et al., 2022)	TT-RS-ST-DL	Stochastic; Interval-based arrival rate	SPN, RSN	GUROBI
This paper	TT-RS-ST-DL	Stochastic; Scenario-based TDOD matrix; vehicle selection behavior	TLV, RSN	GUROBI

Approach: train timetable (TT); stop-skipping (SS); short-turning strategy (ST); rolling stock (RS); single line (SL); double-track line (DL). **Passenger Demand Representation:** time-dependent origin–destination matrix (TDOD matrix); without consideration of passenger demand (NULL). **Objective:** passenger travel time (PTT); passenger satisfaction (PS); train loading variation (TLV); headway deviation (HD); rolling stocks number (RSN); operating cost (OC); passenger waiting number (PWN); passenger waiting time (PWT); fixed cost (FC); energy consumption (EC); stranded passengers number (SPN). **Solution methods:** general-purpose solver (Solver), such as CPLEX, GUROBI; Decomposition-based algorithm (DB); Simulated Annealing algorithm (SA); Genetic algorithm (GA); Lagrangian relaxation- based algorithm (Lagrangian); variable neighborhood search (VNS).

The passenger uncertainty affects the coupling relationship between passengers and train services, and increase the complexity of the train timetabling problem. With these concerns, the train timetable optimization with deterministic passenger flow may no longer be suitable for different scenarios, particularly for instances of large passenger flow (Gong et al., 2021). To address the knowledge gap indicated above, this paper studies the train timetabling problem involving uncertain passenger demand and vehicle selection. The main contributions are presented as follows:

1) This paper investigates a robust optimization approach for the train timetabling problem with short-turning strategy. Due to the uncertainty of passenger demand volume and vehicle selection behavior, the scenario-based passenger demand scenarios representation is introduced by discretizing the involved time horizon. We use the proportion of passengers who take the short-turning train services to the last station of the short-turning region and transfer to the full-length train services to their destination stations, to describe the passenger vehicle selection under short-turning strategy, which is considered in the robust train timetabling problem.

2) A robust train timetabling optimization model is formulated that simultaneously integrates short-turning strategy and rolling stock circulation, in which the objective function is to reduce the load factor variation. The rolling stock mixed application mode, which means the trains of short-turning services can share rolling stocks with the trains of full-length services, has been considered in the train timetabling problem with short-turning strategy for the first time in this study.

3) The proposed methodology is applied to optimize the train timetable of Xi’an Metro

Line 3, China. The results indicate that the robust train timetable can more effectively satisfy multi-scenario passenger demand than the satisfactory train timetable generated by independent optimization of each demand scenario. It can effectively improve the coupling degree between passengers and train services by reducing train capacity oversaturation and waste (see Appendix A), and decrease the number of stranded passengers. In addition, we find the vehicle selection behavior (take the short-turning train services and transfer to the full-length train services) will transform some passengers from the full-length train services to the short-turning train services, and reduce the load factor variation between them.

2 Problem statement and notations

2.1 Problem statement

As shown in Fig.2, a bidirectional train line consisting of N stations in the metro system is investigated in this study. The downward direction denotes the direction that train operates from depot P to depot M, and stations are numbered from 1 to N. In contrast, the opposite direction is defined as upward operations. Take the downward direction as an example, depot P and depot M are employed by the trains of full-length services connected with station 1 and station N, depot A and depot B are used by the trains of short-turning services connected with station a and station b. A downward train of short-turning services departs from depot A to depot B or turns around at depot B. In contrast, a downward train of full-length services departs from depot P to depot M or turns around at depot M. Train operations in the upward direction are similar to the above.

This study investigates the train timetabling problem involving short-turning strategy and rolling stock circulation plan. The train timetabling problem determines train arrival times, departure times, stations' dwell times, and sections' running times. The short-turning strategy decides which two stations to be the short-turning station and which train takes on short-turning services. The rolling stock circulation plan indicates which train services the same train undertakes to improve train utilization.

In most existing studies about short-turning strategy, there are assumptions that only train services of the same type can be connected, which means that the trains of short-turning services cannot share rolling stocks with the trains of full-length services (Zhu et al., 2022). This mode is called rolling stock independent application mode (**Independent-mode**) in this study. In reality, the full-length and short-turning train services can share the rolling stocks if they have a common turn-back station, such as $a = 1$ or $b = n$ in Fig. 2. In other words, the rolling stock can operate a short-turning train service after finishing a full-length train service, and vice versa, this mode is called rolling stock mixed application mode **(Mixed-mode)** in this study. However, if $a \neq 1$ and $b \neq N$, the rolling stock application mode can only be the Independent-mode.

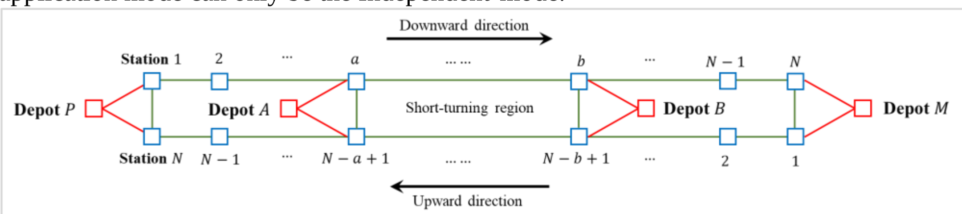


Fig.2 A bidirectional train line with multi deports

In the following, some assumptions are first proposed to formulate the problem.

Assumption 1: All trains have the same capacity, regardless of whether they operate full-length or short-turning services. The running times among adjacent stations are constants, which are not affected by the process of passenger boarding and alighting.

Assumption 2: This study only considers two train service patterns, including one full-length service and one short-turning service, since more service patterns mean greater complexity in the operational organization (Li et al., 2019). The number of full-length train services is proportional to short-turning train services. There are a fixed number of full-length train services between two adjacent short-turning train services. The first train service during the planning period is assumed to be a short-turning service.

Assumption 3: In each scenario, a specific proportion is set for passengers taking the short-turning train services and transfer to the full-length train services, we suppose the "transfer" station is the last station of the short-turning region in the running direction.

2.2 Notations and decision variables

The related symbols used in the formulation are defined as follows.

Table 2 Related symbols used in the formulation

Notation	Description
G	set of passenger scenarios
ψ	index of scenarios, $\psi \in G$
E_ψ	the probability of scenario ψ
D	set of running direction, $D = \{dn, up\}$, dn refers to the downward direction, up refers to the upward direction
x	index of running direction, $x \in D$
S	set of stations, $S = \{1,2,\ldots,N\}$
i, j	index of stations, $i, j \in S$
a, b	index of stations, $a, b \in S$, index of turn-around stations for the downward direction
R^x	set of train services in the running direction x, $R^x = \{1,2,\ldots,\|R^x\|\}$
r, r'	index of train services, $r, r' \in R^x$
t_{start}, t_{end}	starting, ending time of the research time range
K	set of time slots in the operating time period, $K = \{1,2,\ldots,\|K\|\}$
k	index of time slots, $k \in K$
η_r^x	binary constant that refers to whether train service r of the x direction is a short-turning service
c_i^x	train running time between station i and $i+1$ ($i \in S/\{N\}$) on direction x
w_i^x	train dwell time at station i on direction x
h_{min}^x, h_{max}^x	the maximum and minimum headway on direction x
φ_a	binary constant that refers to whether $a = 1$
φ_b	binary constant that refers to whether $b = N$
z_{min}	minimum turn-around time
$p_{i,j,k}^x(\psi)$	number of arriving passengers with journey $i \rightarrow j$ entering in station i of direction x in time slot k in scenario ψ
$p_{i,k}^x(\psi)$	number of arriving passengers entering in station i of direction x in time slot k in scenario ψ
ξ_ψ	proportion of **Type III** (passengers who take the short-turning train services to the last station of the short-turning region, and transfer to the full-length train services to their destination stations.) in Type II in scenario ψ
$\lambda_{i,k}^x(\psi)$	passenger arrival rate of station i of direction x in time slot k in scenario ψ
$\omega_{i,j}^x(\psi)$	proportion of arrival passengers with journey $i \rightarrow j$ in the passengers entering in station i of direction x in scenario ψ
Φ_{max}	maximum load factor

Decision variables used in the formulations.

Table 3 Decision variables used in the formulation

Notation	Description
$d_{r,i}^x$	departure time of train r of direction x at station i
$a_{r,i}^x$	arrive time of train r of direction x at station i
$\kappa_{r,r'}^{dn}$	binary variable that refers to whether upward train service r' is operated by the rolling

	stock that operated downward train service r before
$\kappa_{r',r}^{up}$	binary variable that refers to whether downward train service r is operated by the rolling stock that operated upward train service r' before
$g_{r,i}^{x}(\psi)$	if $r > 1$, number of passengers entering in station i of direction x between the departure times of train service r and $r-1$; if $r = 1$, number of passengers entering in station i of direction x between the departure time of train service 1 and t_{start}.
$f_{r,i}^{x}(\psi)$	number of passengers alight from train service r in station i of direction x in scenario ψ
$b_{r,i}^{x}(\psi)$	number of passengers boarding train service r in station i of direction x in scenario ψ
$w1_{r,i}^{x}(\psi)$	number of passengers waiting in the platform of the station i of the direction x before train service r left in scenario ψ
$w2_{r,i}^{x}(\psi)$	number of passengers remain in the platform of the station i of the direction x after train service r left in scenario ψ
$v_{r,i}^{x}(\psi)$	number of in-vehicle passengers of train service r between stations i and $i+1$ ($i \in S/\{N\}$) of the direction x in scenario ψ, not considering passenger vehicle selection for the full-length and short-turning train services.
$v2_{r,i}^{dn}(\psi)$	number of in-vehicle passengers of train service r between stations i and $i+1$ ($i \in S/\{N\}$) of the direction x in scenario ψ, considering passenger vehicle selection for the full-length and short-turning train services.
$\theta_{r,i}^{x}(\psi)$	load factor value of train service r between stations i and $i+1$ ($i \in S/\{N\}$) of direction x in scenario ψ
$\chi_{r,i}^{x}(\psi)$	risk degree of train load factor over the maximum load factor, for train service r between stations i and $i+1$ ($i \in S/\{N\}$) of direction x in scenario ψ

2.3 Representation of passenger demand and vehicle selection with short-turning strategy

To represent the dynamic feature of passenger demand within one scenario, we adopt the TDOD matrix to describe the specific information of passengers, including their origins, destinations and arrival times at individual origin stations. The uncertainty of passenger demand volume and vehicle selection behavior can be reflected across different stochastic scenarios, where time-dependent passenger demand varies randomly, and the related parameters about vehicle selection are set differently. Let G be the set of passenger scenarios and $E_{\psi} > 0$ be the probability of scenario $\psi \in G$ with $\sum_{\psi \in G} E_{\psi} = 1$.

Note that the research time range $[t_{start}, t_{end}]$ is split into K time slots with the splitting time instants $t_1, t_2, \dots, t_{k-1}, t_k, \dots, t_{K-1}$, and $t_0 = t_{start}$, $t_K = t_{end}$. For scenario $\psi \in G$, $p_{i,j,k}^{x}(\psi)$ denotes the number of arriving passengers with journey $i \rightarrow j$ entering in the station i of the direction x in the time slot k, that is, the time interval $[t_{k-1}, t_k)$. Then, the corresponding scenario-based TDOD matrix for a bidirectional urban rail transit line with N stations can be represented by

$$\begin{pmatrix} 0 & p_{1,2,k}^{x}(\psi) & p_{1,3,k}^{x}(\psi) & \cdots & p_{1,N,k}^{x}(\psi) \\ 0 & 0 & p_{2,3,k}^{x}(\psi) & \cdots & p_{2,N,k}^{x}(\psi) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & p_{N-1,N,k}^{x}(\psi) \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

And $p_{i,k}^{x}(\psi)$ denotes the number of passengers entering in the station i of the direction x in time interval $[t_{k-1}, t_k)$, can be calculated by

$$p_{i,k}^{x}(\psi) = \sum_{j=i+1}^{N} p_{i,j,k}^{x}(\psi) \quad \forall\, i \in [1, N-1].$$

The passenger arrival rate of station i of the direction x can be written as:

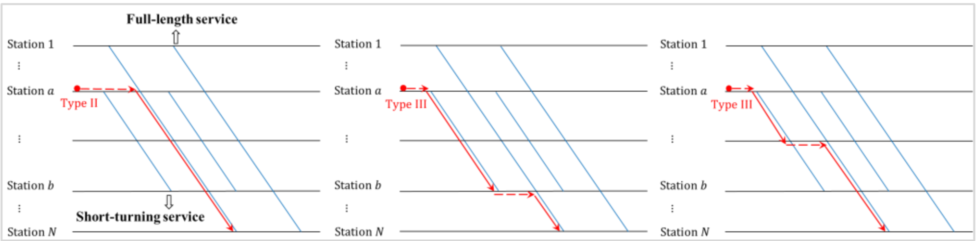
$$\lambda_{i,k}^{x}(\psi) = p_{i,k}^{x}(\psi)/(t_k - t_{k-1}).$$

Then, $\omega_{i,j}^{x}(\psi)$ denotes the proportion of arrival passengers with journey $i \rightarrow j$ in the passengers entering in station i of direction x,

$$\omega_{i,j}^{x}(\psi) = \frac{\sum_{k=1}^{|K|} p_{i,j,k}^{x}(\psi)}{\sum_{k=1}^{|K\}} p_{i,k}^{x}(\psi)} \ \ \forall\, \psi \in G, x \in D\ j \in [i+1, N].$$

As shown in Fig.1, in the short-turning region, some passengers can't reach their destination stations directly by the short-turning train services. Take the downward direction as an example, for the passengers entering in the stations $(a, a+1, \ldots, b-1)$, they can't reach to the stations $(b+1, b+2, \ldots, N)$ directly by the short-turning train services. In the short-turning region, we divide $p_{i,k}^{x}(\psi)$ into the following two types of passengers: 1) **Type I**: they can reach their destination stations directly by the short-turning or full-length train services, are denoted as $p1_{i,k}^{x}(\psi)$. 2) **Type II**: they can't reach their destination stations directly by the short-turning train services, in other words, they must take the full-length train services, are denoted as $p2_{i,k}^{x}(\psi)$.

The existing research about short-turning strategy mostly assumed that passengers would choose the direct trains to their destinations (Li et al., 2019;Zhu et al., 2022). Type II must wait on the platform and take the full-length train services to reach their destination stations. In reality, some passengers of Type II will take the short-turning train services to the one station of the short-turning region, and transfer to the full-length train services to their destination stations. This study defines passengers with this transfer behavior as **Type III**. Take the passengers with journey $a \rightarrow N$ in the downward direction as an example, as shown in Fig.3(a), Type II must wait on the platform of Station a and take the full-length train services to Station N. Considering the vehicle selection behavior, Type III can firstly take the short-turning train services to the last station (Fig.3(b)) or other stations (Fig.3(c)) in the short-turning region, and transfer to the full-length train services to Station N. The unique vehicle selection behavior will change the coupling relationship between passengers and train services, and affect the train loading factors. For scenario $\psi \in G$, ξ_{ψ} denotes the proportion of Type III in Type II. Therefore, this study uses the scenario-based representation method to capture these two features. Each scenario is assumed to correspond to a distinguishing day and a specific vehicle selection proportion.



(a) Type II (b) Type III (transfer in Station b) (c) Type III (not transfer in Station b)

Fig. 3 Demonstration of vehicle selection behavior for Type II and Type III

3 Methodology

To reduce the number of decision variables, we add dummy running segments to short-turning services at stations outside the region of short-turning service. As shown in Fig. 4, those dummy departure times of short-turning services from the station a to station 1 are extended from the departure time of station a, while those times from station b to station N are extended from the departure time of short-turning train services at station b.

Then, η_{r}^{x} represents whether train service r of the x direction is a short-turning service, which is given as

$$\eta_r^x = \begin{cases} 1, \text{if train service } r \text{ of the } x \text{ direction is a short} - \text{turning service} \\ 0, \text{otherwise} \end{cases} \quad \forall\, x \in D, r \in R^x \tag{1}$$

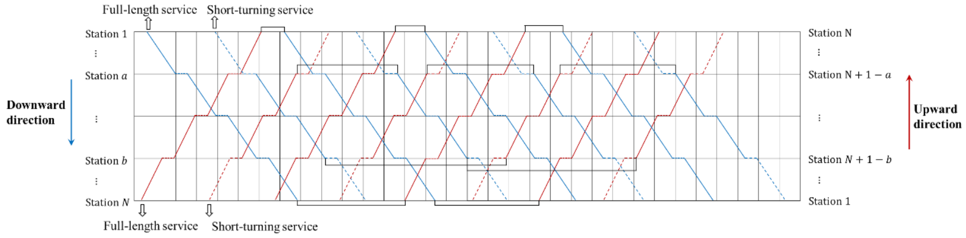


Fig. 4 Demonstration of train operations with short-turning strategy

3.1 Train operation constraints

First, the arrival and departure times of train service from the first station in the running direction is bounded within the research time range, we have

$$t_{start} \le d_{r,1}^x \le t_{end}, \qquad \forall\, x \in D, r \in R^x \tag{2}$$

Constraints (3–4) are built to determine the arrival and departure times of other stations:

$$a_{r,i}^x = d_{r,1}^x + \sum_{j=1}^{i-1} c_j^x + \sum_{j=2}^{i} w_j^x, \qquad \forall\, x \in D, r \in R^x, i \in S/\{1\} \tag{3}$$

$$d_{r,i}^x = a_{r,i}^x + w_i^x, \qquad \forall\, x \in D, r \in R^x, i \in S/\{1\} \tag{4}$$

Moreover, headways should satisfy the maximum and minimum headway constraints for passenger satisfaction and safety, as follows:

$$h_{min}^x \le d_{r,1}^x - d_{r-1,1}^x \le h_{max}^x, \qquad \forall\, x \in D, r \in R^x/\{1\} \tag{5}$$

$$h_{min}^x \le d_{1,1}^x \le h_{max}^x, \qquad \forall\, x \in D \tag{6}$$

$$h_{min}^x \le t_{end} - d_{|R^x|,1}^x \le h_{max}^x, \qquad \forall\, x \in D \tag{7}$$

3.2 Rolling stock circulation constraints

Rolling stock can continue to serve another service in the opposite direction or return to the depot when finishing one train service. Similarly, train service can be operated by the rolling stock in the depot or the one that has finished the previous train service. Then, we introduce the following two binary variables to demonstrate the connection relationships between two train services of different directions:

$\kappa_{r,r'}^{dn}$: if upward train service r' is operated by the rolling stock that operated downward train service r before, $\kappa_{r,r'}^{dn} = 1$; otherwise, $\kappa_{r,r'}^{dn} = 0$.

$\kappa_{r',r}^{up}$: if downward train service r is operated by the rolling stock that operated upward train service r' before, $\kappa_{r',r}^{up} = 1$; otherwise, $\kappa_{r',r}^{up} = 0$.

For the downward train services connected with upward train services, constraints are shown as follows:

$$\sum_{r=1}^{R^{dn}} \kappa_{r,r'}^{dn} \le 1, \qquad \forall r' \in R^{up} \tag{8}$$

$$\sum_{r'=1}^{R^{up}} \kappa_{r,r'}^{dn} \le 1, \qquad \forall r \in R^{dn} \tag{9}$$

Constraint (8) denotes that if $\sum_{r=1}^{R^{dn}} \kappa_{r,r'}^{dn} = 1$, the upward train service r' can only be operated by one rolling stock that has finished a downward service; otherwise, the upward train service r' is operated by the train from the depot. Constraint (9) denotes that if $\sum_{r'=1}^{R^{up}} \kappa_{r,r'}^{dn} = 1$, only one upward train service can be operated by the rolling stock which has finished the downward train service r; otherwise, the rolling stock of train service r will return to the depot and don't operate any train service. The similar constraints for the upward train services connected with downward train services is shown in Constraints (10 11).

$$\sum_{r=1}^{R^{dn}} \kappa_{r',r}^{up} \leq 1, \qquad \forall r' \in R^{up} \tag{10}$$

$$\sum_{r'=1}^{R^{up}} \kappa_{r',r}^{up} \leq 1, \qquad \forall r \in R^{dn} \tag{11}$$

Therefore, we introduce two binary constants to judge whether $a = 1$ or $b = N$.

$$\varphi_a = \begin{cases} 1, & \text{if } a = 1 \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

$$\varphi_b = \begin{cases} 1, & \text{if } b = N \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

And the constraints are shown as follows:

$$\eta_r^{dn} - \eta_{r'}^{up} \leq M\left(1 - \kappa_{r,r'}^{dn}\right) + \varphi_b \quad \forall r \in R^{dn}, r' \in R^{up} \tag{14}$$

$$\eta_r^{dn} - \eta_{r'}^{up} \geq -M\left(1 - \kappa_{r,r'}^{dn}\right) - \varphi_b \quad \forall r \in R^{dn}, r' \in R^{up} \tag{15}$$

$$\eta_{r'}^{up} - \eta_r^{dn} \leq M\left(1 - \kappa_{r',r}^{up}\right) + \varphi_a \quad \forall r \in R^{dn}, r' \in R^{up} \tag{16}$$

$$\eta_{r'}^{up} - \eta_r^{dn} \geq -M\left(1 - \kappa_{r',r}^{up}\right) - \varphi_a \quad \forall r \in R^{dn}, r' \in R^{up} \tag{17}$$

Constraints (14-15) denote that if $\varphi_b = 0$ and $\kappa_{r,r'}^{dn} = 1$, then $\eta_r^{dn} - \eta_{r'}^{up} = 0$, which means that if upward train service r' is operated by the rolling stock that operated downward train service r before, train service r' and r must be the same type (full-length or short-turning); if $\varphi_b = 1$ and $\kappa_{r,r'}^{dn} = 1$, then $-1 \leq \eta_r^{dn} - \eta_{r'}^{up} \leq 1$, which means that the connection relationship between train service r' and r has no requirement on these two trains' type, the rolling stock application mode can be the Mixed-mode. The similar constraints about $\kappa_{r',r}^{up}$ are shown in Constraints (16-17).

Moreover, if two train services in different directions can be connected, the rolling stock operating those two train services needs to satisfy the turn-around time constraint in the depot. For downward train services connect with upward train services, the depots for train circulation are connected with station b or station N in the upward direction, and the constraints for turn-around time are presented as follows:

$$\left(a_{r',1}^{up} - d_{r,N}^{dn}\right)(1 - \eta_r^{dn}) + \left(a_{r',N-b+1}^{up} - d_{r,b}^{dn}\right)\eta_r^{dn} \geq z_{min} - M\left(1 - \kappa_{r,r'}^{dn}\right), \forall r \in R^{dn}, r' \in R^{up} \tag{18}$$

where z_{min} is the minimum turn-around time, $a_{r',1}^{up} - d_{r,N}^{dn}$ represents the time interval between upward train service r' arriving at station 1 and downward train service r departing from station N. Similarly, $a_{r',N-b+1}^{up} - d_{r,b}^{dn}$ the time interval between upward train service r' arriving at station $N - b + 1$ and downward train service r departing from station b. If $\kappa_{r,r'}^{dn} = 1$ and $\eta_r^{dn} = 1$, downward train r is a short-turning service, so $a_{r',N-b+1}^{up} - d_{r,b}^{dn}$ must satisfies the minimum turn-around time; while if $\kappa_{r,r'}^{dn} = 1$ and $\eta_r^{dn} = 0$, downward train r is a full-length service, so $a_{r',1}^{up} - d_{r,N}^{dn}$ must satisfies the minimum turn-around time. The similar constraint about upward train services connect with downward train services is shown in Constraint (19).

$$\left(a_{r,1}^{dn} - a_{r,N}^{up}\right)\left(1 - \eta_{r'}^{up}\right) + \left(a_{r,a}^{dn} - d_{r',N-a+1}^{up}\right)\eta_{r'}^{up} \geq z_{min} - M\left(1 - \kappa_{r',r}^{up}\right), \forall r \in R^{dn}, r' \in R^{up} \tag{19}$$

3.3 Passenger demand constraints

Passenger demand is a significant factor for train timetable optimization problems, since the number of train services and the headways between train services highly depend on this factor (Wang et al., 2018). In real operations, the number of in-vehicle passengers will change dynamically with the change in headways. We next intend to formulate the specific constraints to describe this process. First, the number of passengers waiting for the train service r in the station i of the direction x can be written as

$$g^x_{r,i}(\psi) = \begin{cases} \int_{t_{start}}^{d^x_{1,i}} \lambda^x_{i,k}(\psi)dk & \text{if} \quad r = 1 \\ \int_{d^x_{r-1,i}}^{d^x_{r,i}} \lambda^x_{i,k}(\psi)dk & \text{if} \quad r \in R^x/\{1\} \end{cases} \qquad \forall\, \psi \in G, x \in D, i \in S \tag{20}$$

where if $r > 1$, $g^x_{r,i}(\psi)$ refers to the number of passengers entering in station i of direction x between the departure times of train service r and $r-1$; if $r = 1$, $g^x_{r,i}(\psi)$ refers to the number of passengers entering in station i of direction x between the departure time of train service 1 and t_{start}.

The number of passengers alight from the train service r in the station i of the direction x is denoted as $f^x_{r,i}(\psi)$, we have

$$f^x_{r,i}(\psi) = f1^x_{r,i}(\psi) + f2^x_{r,i}(\psi), \quad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{21}$$

$$f1^x_{r,i}(\psi) = \sum_{j=1}^{i-1} \left(b1^x_{r,j}(\psi) * \omega1^x_{j,i}(\psi)\right) \qquad \forall\, \psi \in G, r \in R^x, i \in S \tag{22}$$

$$f2^x_{r,i}(\psi) = \sum_{j=1}^{i-1} \left(b2^x_{r,j}(\psi) * \omega2^x_{j,i}(\psi)\right) \qquad \forall\, \psi \in G, r \in R^x, i \in S \tag{23}$$

where $f1^x_{r,i}(\psi)$ and $f2^x_{r,i}(\psi)$ refer to the number of Type I, Type II alighting from the train service r in the station i of the direction x, respectively; $b1^x_{r,j}(\psi)$ and $b2^x_{r,j}(\psi)$ refer to the number of Type I, Type II boarding the train service r in the station i of the direction x, respectively, can be calculated by

$$f1^x_{r,i}(\psi) = g1^x_{r,i}(\psi) \qquad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{24}$$

$$b2^x_{r,i}(\psi) = \begin{cases} w1^x_{r,i}(\psi) - g1^x_{r,i}(\psi) & \eta_r = 0 \\ 0 & \eta_r = 1 \end{cases} \quad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{25}$$

$$b^x_{r,i}(\psi) = w1^x_{r,i}(\psi) * (1 - \eta^x_r) + g1^x_{r,i}(\psi) * \eta^x_r \qquad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{25}$$

$$w1^x_{r,i}(\psi) = \begin{cases} g1^x_{r,i}(\psi) + g2^x_{r,i}(\psi) & r = 1 \\ g1^x_{r,i}(\psi) + g2^x_{r,i}(\psi) + w2^x_{r-1,i}(\psi) & r \in R^x/\{1\} \end{cases} \quad \forall\, \psi \in G, x \in D, i \in S \tag{26}$$

$$w2^x_{r,i}(\psi) = w1^x_{r,i}(\psi) - b^x_{r,i}(\psi) \qquad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{27}$$

where $w1^x_{r,i}(\psi)$ refers to the number of passengers waiting in the platform of the station i of the direction x before train service r left; $w2^x_{r,i}(\psi)$ is the number of passengers remain in the platform of the station i of the direction x after train service r left.

Therefore, we can calculate the number of in-vehicle passengers of train service r between stations i and $i+1$ ($i \in S/\{N\}$) of the direction x, which is denoted as $v^x_{r,i}(\psi)$:

$$v^x_{r,i}(\psi) = \begin{cases} b^x_{r,i}(\psi) & i = 1, \\ v^x_{r,i-1}(\psi) - f^x_{r,i}(\psi) + b^x_{r,i}(\psi), & i > 1, \end{cases} \quad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{28}$$

As shown in Section 2.3, some passengers (Type III) of Type II will take the short-turning train services to the last station of the short-turning region, and wait for the full-length train services to their destination stations. Therefore, we should modify the above constraints about the coupling relationship between passengers and train services considering the uncertainty of passenger vehicle selection. For the short-turning train services, the number of in-vehicle passengers will increase; in contrast, the number of passengers for full-length train services will reduce. Therefore, this study should modify Constraint (28). Take the downward direction as an example, which can be written as follows:

$$v2_{r,i}^{dn}(\psi)=\begin{cases} v_{r,i}^{dn}(\psi)+\sum_{j=1}^{i}(\xi_\psi * b2_{r,i}^{dn}(\psi)) & \eta_r^{dn}=1 \\ v_{r,i}^{dn}(\psi)-\sum_{j=1}^{i}(\xi_\psi * b2_{r-1,i}^{dn}(\psi) * \eta_{r-1}^{dn}) & \eta_r^{dn}=0 \end{cases} \quad \forall\, \psi \in G, r \in R^{dn}, i \in [a, b-1] \tag{29}$$

$$v2_{r,i}^{dn}(\psi)=\begin{cases} 0 & \eta_r^{dn}=1 \\ v_{r,i}^{dn}(\psi) & \eta_r^{dn}=0 \end{cases} \quad \forall\, \psi \in G, r \in R^{dn}, i \notin [a, b-1] \tag{30}$$

where $v2_{r,i}^{dn}(\psi)$ refers to the number of in-vehicle passengers of train service r between stations i and $i+1$ ($i \in S/\{N\}$) of the direction x, considering passenger vehicle selection for the full-length and short-turning train services.

Constraint (29) is for the sections in the short-turning region, if train r is short-turning service, $\eta_r^{dn}=1$, $v_{r,i}^{dn}(\psi)$ should add the number of Type III boarding at and before station i, which can be calculated by $\sum_{j=1}^{i}(\xi_\psi * b2_{r,i}^{dn}(\psi))$; if train r is full-length service, $\eta_r^{dn}=0$, we should judge whether train $r-1$ is full-length service. If train $r-1$ is full-length service, $v_{r,i}^{dn}(\psi)$ should reduce the number of Type III boarding at train $r-1$; otherwise, $v2_{r,i}^{dn}(\psi)$ is equal to $v_{r,i}^{dn}(\psi)$. Constraint (30) is for the sections not in the short-turning region, if train r is short-turning service, $v2_{r,i}^{dn}(\psi)$ equals 0. If train r is full-length service, $v2_{r,i}^{dn}(\psi)$ is equal to $v_{r,i}^{dn}(\psi)$. The similar constraints for the upward direction are shown in Constraints (31)-(32).

$$v2_{r,i}^{up}(\psi)=\begin{cases} v_{r,i}^{up}(\psi)+\sum_{j=1}^{i}(\xi_\psi * b2_{r,i}^{up}(\psi)) & \eta_r^{up}=1 \\ v_{r,i}^{up}(\psi)-\sum_{j=1}^{i}(\xi_\psi * b2_{r-1,i}^{up}(\psi) * \eta_{r-1}^{up}) & \eta_r^{up}=0 \end{cases} \quad \forall\, \psi \in G, r \in R^{up}, i \in [N-b+1, N-a] \tag{31}$$

$$v2_{r,i}^{up}(\psi)=\begin{cases} 0 & \eta_r^{up}=1 \\ v_{r,i}^{up}(\psi) & \eta_r^{up}=0 \end{cases} \quad \forall\, \psi \in G, r \in R^{up}, i \notin [N-b+1, N-a] \tag{32}$$

The load factor of the train service r of the direction x between stations i and $i+1$ can be written as:

$$\theta_{r,i}^{x}(\psi)=v2_{r,i}^{up}(\psi)/C, \qquad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{33}$$

Similar to (Gong et al., 2021), the train load factor must be no more than the maximum load factor during the operational period in each scenario, as formulated below:

$$\theta_{r,i}^{x}(\psi) \le \Phi_{max}, \qquad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{34}$$

However, it's difficult for every scenario to satisfy constraint (34) in the robust solution, so we introduce the risk coefficient $\chi_{r,i}^{x}(\psi)$, which refers to the risk degree of train load factor over the maximum load factor, and modify constraint (34) to constraint (35):

$$\theta_{r,i}^{x}(\psi) \le \Phi_{max}+\chi_{r,i}^{x}(\psi), \qquad \forall\, \psi \in G, x \in D, r \in R^x, i \in S \tag{35}$$

$$\chi_{r,i}^{x}(\psi) \ge 0, \qquad \forall \psi \in G, x \in D, r \in R^x, i \in S \tag{36}$$

3.4 Objective Function

In this study, the goals of the train timetabling are to reduce the load factor variation and improve the utilization of the rolling stocks. We take the utilization of the rolling stocks as an ε-constraint (Rabiee et al., 2014) and focus on reducing the load factor variation. The ε-constraint is expressed as

$$|R^{dn}|+|R^{up}|-\sum_{r=1}^{R^{dn}}\sum_{r'=1}^{R^{up}}(\kappa_{r,r'}^{dn}+\kappa_{r',r}^{up}) \le \varepsilon \tag{37}$$

where $\sum_{r=1}^{R^{dn}}\sum_{r'=1}^{R^{up}}(\kappa_{r,r'}^{dn}+\kappa_{r',r}^{up})$ refers to the connection number of different directions' train services, so the left side of Constraint (37) the total rolling stock number of the train timetable, the right side restricts the total rolling stock number to not exceed a certain level. Hence, the model transforms the dual objective problem into a single objective problem, which focuses on reducing the load factor variation under limited transportation resources. In this study, the objective function is formulated as follows:

$$\partial 1_{r,i}^{x}(\psi) = max\{0, \theta_{r,i}^{x}(\psi) - \overline{\Phi}\} \tag{38}$$

$$\partial 2_{r,i}^{x}(\psi) = max\{0, \underline{\Phi} - \theta_{r,i}^{x}(\psi)\} \tag{39}$$

$$min\, Z(\psi) = \sum_{x\in D}\sum_{r\in R^x}\sum_{i\in S}\left(\delta 1 * \partial 1_{r,i}^{x}(\psi) + \delta 2 * \partial 2_{r,i}^{x}(\psi) \; + \; \delta 3 * \chi_{r,i}^{x}(\psi)\right) \tag{40}$$

where $\overline{\Phi}$ and $\underline{\Phi}$ refer to the upper and lower bounds of the "comfortable" load factor, respectively. $\partial 1_{r,i}^{x}(\psi, v)$ and $\partial 2_{r,i}^{x}(\psi, v)$ are the load factor variation values, which refer to the degree of train capacity oversaturation and waste, respectively. if $\theta_{r,i}^{x}(\psi)$ is within $[\underline{\Phi}, \overline{\Phi}]$, $\partial 1_{r,i}^{x}(\psi) = 0, \partial 2_{r,i}^{x}(\psi) = 0$; if $\theta_{r,i}^{x}(\psi)$ is above $\overline{\Phi}$, $\partial 1_{r,i}^{x}(\psi) > 0, \partial 2_{r,i}^{x}(\psi) = 0$; if $\theta_{r,i}^{x}(\psi)$ is less than $\underline{\Phi}$, $\partial 1_{r,i}^{x}(\psi) = 0$, $\partial 2_{r,i}^{x}(\psi) > 0$. The objective function considers reducing the degree of train capacity oversaturation and waste, so as to reduce the load factor variation. If $\theta_{r,i}^{x}(\psi)$ is above Φ_{max}, there are some passengers stranded in the platform, and the number of stranded passengers is the value of $\chi_{r,i}^{x}(\psi)$ multiplied by train standard capacity. $\delta 1$, $\delta 2$ and $\delta 3$ are the corresponding weight coefficients to reach a balance between train capacity oversaturation, waste and risk. As discussed above, the robust optimization model for train timetabling and the short-turning strategy is shown as follows:

$$\min Z = \sum_{\psi\in G}(Z(\psi) * E_{\psi}) \tag{41}$$

$$\text{s. t. Constraints: } (1) - (33), (35) - (40).$$

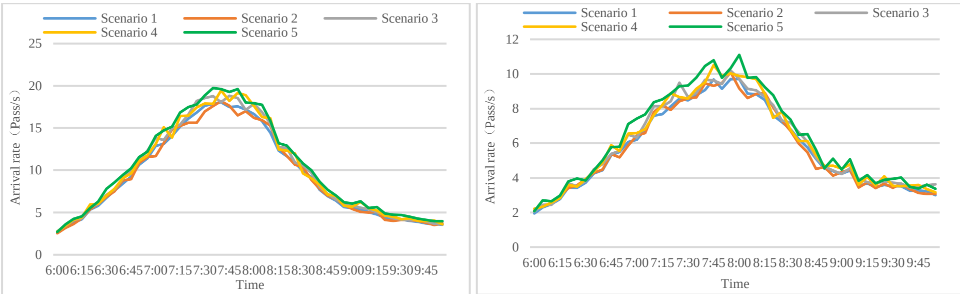
4 Case Study

A case study of the Xi'an Metro Line 3 in Shaanxi, China, in 2021 is used to test the proposed method. The Xi'an Metro Line 3 consists of 26 stations. The downward direction is defined as the train operating from Bonded Zone Station (the 1st station in the downward direction) to Yuhuazhai Station (the 26th station in the downward direction). This area is also the full-length services region. The short-turning services region is between Xianghuwan Station (the 6th station in the downward direction) and Yuhuazhai Station. These three stations are connected with the depots. Note that different service types (full-length, short-turning) of downward and upward train services can be connected (share the rolling stocks) at Yuhuazhai Station.

The period from 7:00 a.m. to 9:00 a.m. is considered the planning horizon, and 24 train services are operated in each direction. The maximum rolling stocks number is set to 30, which means there are at least 18 train services connections. The maximum and minimum headways are 360 s and 120 s, respectively. Each station's standard running and dwell times are adopted in this study according to the original timetable, which and available in the public domain, as announced by the Xi'an metro. The minimum turn-around time in these three depots is 180 s. Moreover, the standard and maximum capacity of train service are 1380 and 2070 persons (the maximum load factor is 1.5), respectively, for both short-turning and full-length services. The optimization model is implemented using MATLAB R2020a with Intel Core 2.40 GHz CPUs computer, and the Yalmip toolbox with Gurobi version 9.5.0.

Firstly, we set a passenger scenario set named G_1, which includes five scenarios (Scenario 1-5). The total number of passengers is 225137 (Scenario 1), 224674 (Scenario

2), 233418 (Scenario 3), 236718 (Scenario 4), and 247389 (Scenario 5), respectively. We set the probability of each scenario equally as 0.2 (i.e., E_{ψ}= 0.2, $\psi \in G_1$). In each scenario of G_1, we do not consider the uncertainty of passenger vehicle selection. For these scenarios, the total demand of all stations in the downward and upward directions is shown in Fig. 5. It can be seen that passenger demand from 7:00 to 9:00 maintains a relatively higher level than that from 6:00 to 7:00 and 9:00 to 10:00, and shows a decreasing trend after 8:00.



(a) downward direction (b) upward direction

Fig. 5 Passenger demand of the Xi'an Metro Line 3.

Then, we set another passenger scenario set, named G_2, which includes six scenarios (Scenario 6-11). Different from G_1, we consider the uncertainty of passenger demand and vehicle selection in G_2. In Scenario 6,7,8, the spatiotemporal passenger demands are consistent with that of Scenario 2 in G_1, and ξ_{ω} set to 0.1,0.2 and 0.3, respectively. In Scenario 9,10,11, the spatiotemporal passenger demands are consistent with that of Scenario 5 in G_1, and ξ_{ω} is set to 0.1,0.2 and 0.3, respectively.

Three simulation experiments are conducted in this section. Experiment 1 compared the results of different cases with short-turning services under the passenger scenario set G_1. Experiment 2 analyzes the effectiveness of the robust optimization considering the uncertainty of the passenger demand, under the passenger scenario set G_1. Experiment 3 analyzes the effectiveness of the robust optimization considering the uncertainty of the passenger demand and vehicle selection, under the passenger scenario set G_2.

4.1 Experiment 1: Optimal cases comparison

In Experiment 1, we compared optimal cases, including two all full-length services cases (Cases I and II) and three short-turning services cases (Cases III, IV and V). For the two full-length services cases, Case I uses the fixed departure frequency strategy (The interval between two adjacent trains is the same), and Case II uses the flexible departure frequency strategy (The interval between two adjacent trains is different). For short-turning services strategies, the proportion of full-length services and short-turning services is set to 3 (Case III), 2 (Case IV), and 1 (Case V), in which the proportion of 2 indicates that the two train services after a short-turning service are full-length services. The rolling stock application mode is the Mixed-mode in the optimal timetables with the short-turning services strategy. The optimization results of those five cases are listed in Table 4.

Table 4. Comparison results with different strategies.

Case	Objective	Oversaturation	Waste	Risk	RSN
I	291.86	56.69	415.29	5.50	28
II	256.00	40.32	406.29	2.50	30
III	218.95	46.23	340.75	4.66	30

IV	231.94	60.60	315.79	2.68	29
V	197.46	52.80	283.96	5.33	28

Oversaturation: degree of train capacity oversaturation; **Waste**: degree of train capacity waste; **Risk**: degree of train capacity risk; **RSN**: number of rolling stocks needed.

The optimization results of these five cases are listed in Table 4. Case V is superior to other cases concerning the objective values and the rolling stock number. Compared with Case I, the degrees of train capacity oversaturation and waste decrease in Case II. Because the flexible departure frequency strategy can adjust the allocation of train capacity in the time dimension according to the passenger demand distribution, although the full-length service can serve all stations, it wastes the train capacity in these sections with small passenger demand. Compared with Case II, the objective values of Case III, IV, and V reduce. The short-turning service can adjust the allocation of train capacity in the space dimension, to reduce the waste of train capacity in these sections. In addition, the trains of short-turning services take a shorter time for circulation, so it only needs 28 rolling stocks, which are needed 30 in Case II, III, and 29 in Case IV, respectively.

The optimal timetable with 24 train services in both directions obtained from Case V is shown in Fig. 6. The optimal timetables for the other four situations are shown in **Appendix B**. It can be seen that there are 4, 7 and 9 train connections in Bonded Zone Station, Xianghuwan Station, and Yuhuazhai Station, respectively. We also use the Independent-mode to calculate the rolling stock circulation plan under the optimal timetable. As shown in **Appendix C**, there are only eight train connections in Yuhuazhai Station, so it needs 29 rolling stocks. In other words, the Mixed-mode can improve the utilization of rolling stocks because the trains of different service types can share the rolling stocks, and the rolling stock circulation time can reduce.

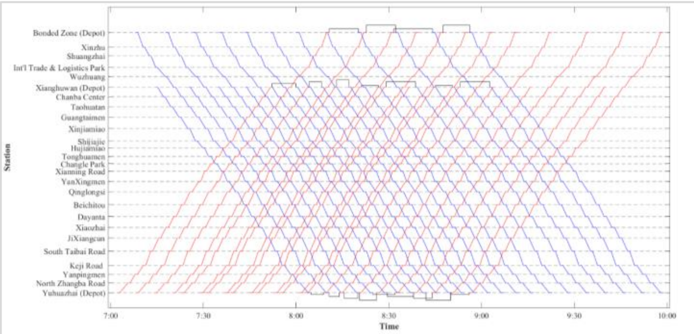


Fig. 6 Optimal train timetable with Case V (Mixed-mode).

4.2 Experiment 2: Uncertainty of the passenger demand

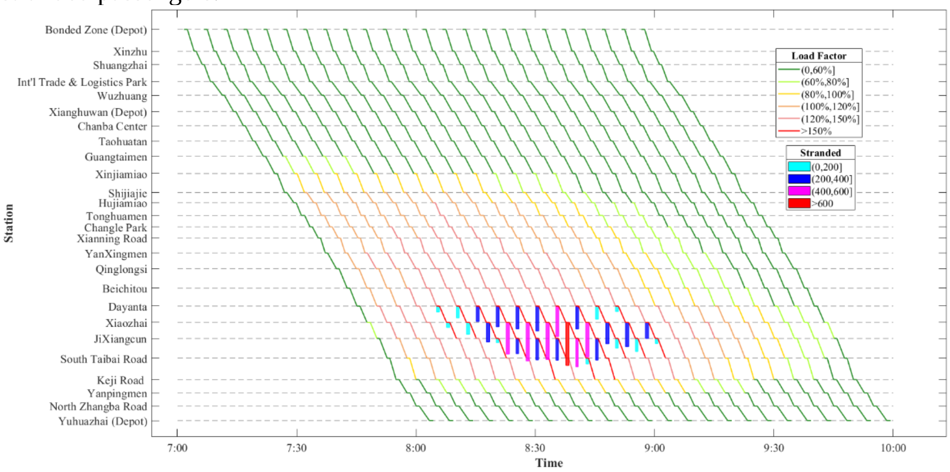
Based on Case V in Experiment 1, this experiment analyzes the effectiveness of the robust optimization considering the uncertainty of the passenger demand under the passenger scenario set G_1. Firstly, each demand scenario's satisfactory (best) train timetable is generated according to passenger demand, denoted as S_1, S_2, …, and S_5, respectively. Secondly, these satisfactory train timetables are implemented in all demand scenarios. Thirdly, the robust train timetable obtained from Case V in Experiment I is implemented on all demand scenarios, denoted as S_{robust}. Note the probability of each scenario in G_1 is 0.2.

The indicators corresponding to the six optimal timetables are shown in Table 5. The results show that if a train timetable is selected among these satisfactory timetables and implemented in all demand scenarios, the average objective value will be larger than for the robust train timetable. It proves that the robust train timetable can more effectively satisfy multi-scenario passenger demand than the satisfactory train timetable generated by independent optimization of each demand scenario.

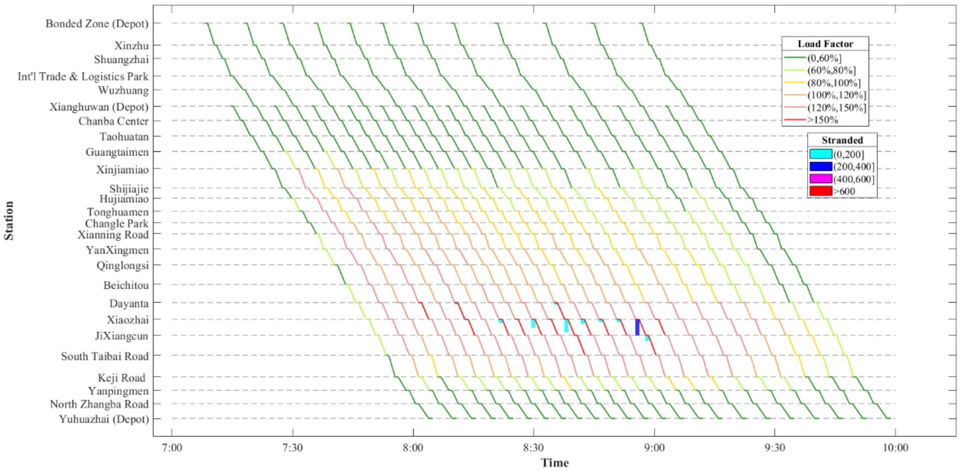
Table 5. Comparison of indicators under different passenger demand scenarios in Experiment 1

Optimal timetable	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average objective
$\boldsymbol{S_1}$	186.15	187.30	203.57	208.45	217.34	200.56
$\boldsymbol{S_2}$	187.81	185.24	204.01	208.37	216.42	200.37
$\boldsymbol{S_3}$	189.27	188.63	198.38	205.07	211.79	198.62
$\boldsymbol{S_4}$	189.98	189.18	202.86	199.65	214.88	199.31
$\boldsymbol{S_5}$	189.58	188.29	200.45	204.65	208.74	198.34
$\boldsymbol{S_{robust}}$	188.13	187.07	199.59	202.70	209.80	197.45

Fig. 7 shows the train load factor and stranded passenger number in the downward direction for the original train timetable and the robust train timetable under Scenario 5. Green means that the section has train capacity waste, and red means that the section has train capacity oversaturation. It can be observed that the train capacity waste in the sections of the zone between Bonded Zone Station and Xianghuwan Station is obvious. The robust train timetable with short-turning strategy can reduce train capacity waste by reducing the train running number in these sections. The number of the sections with train capacity oversaturation and waste reduce from 334 to 238 and 170 to 158, respectively. In addition, the stranded passenger number reduces obviously, and the maximum number of stranded passengers on the platform is reduced from 605 to 233. In conclusion, the robust train timetable can effectively improve the coupling degree between passengers and train services by reducing train capacity oversaturation and waste, and decrease the number of stranded passengers.



(a) Original train timetable



(b) Robust train timetable

Fig. 7 Comparison of train load factor and stranded passenger in the downward direction

4.3 Experiment 3: Uncertainty of the passenger demand and vehicle selection

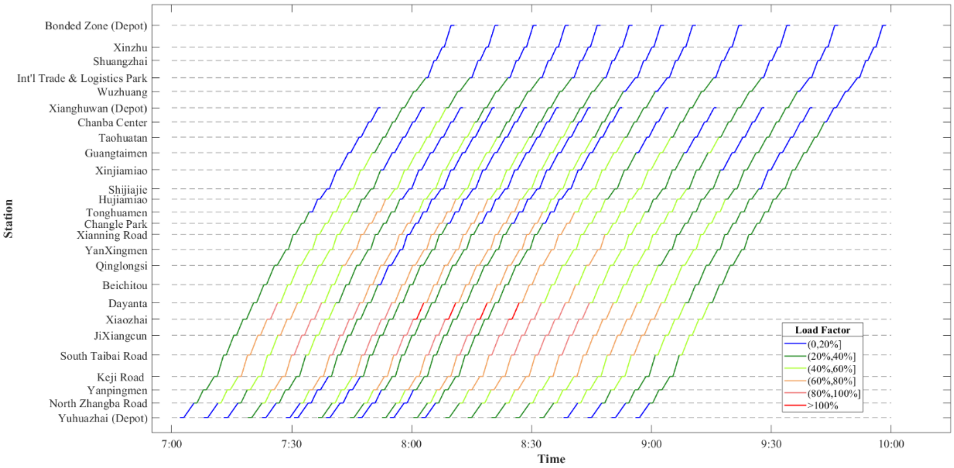
Based on Case V in Experiment 1, this experiment analyzes the effectiveness of the robust optimization considering the uncertainty of the passenger demand and vehicle selection, under the passenger scenario set G_2. Similar to Experiment 2, the satisfactory (best) train timetable of each passenger scenario is generated according to passenger demand and vehicle selection, denoted as S6, S7, …, and S11, respectively, and we calculate the robust train timetable according to the passenger scenarios set G_2, denoted as $S_{robust2}$. These six satisfactory train timetables and the robust train timetable are also implemented in all passenger scenarios of G_2, and the corresponding indicators are shown in Table 6. Note the probability of each scenario in G_2 is 1/6.

The results also show that if a train timetable is selected among these satisfactory timetables and implemented in all demand scenarios, the average objective value will be larger than for the robust train timetable. In other words, the robust train timetable can more effectively satisfy multi-scenario passenger demand than the satisfactory train timetable generated by independent optimization of each demand scenario.

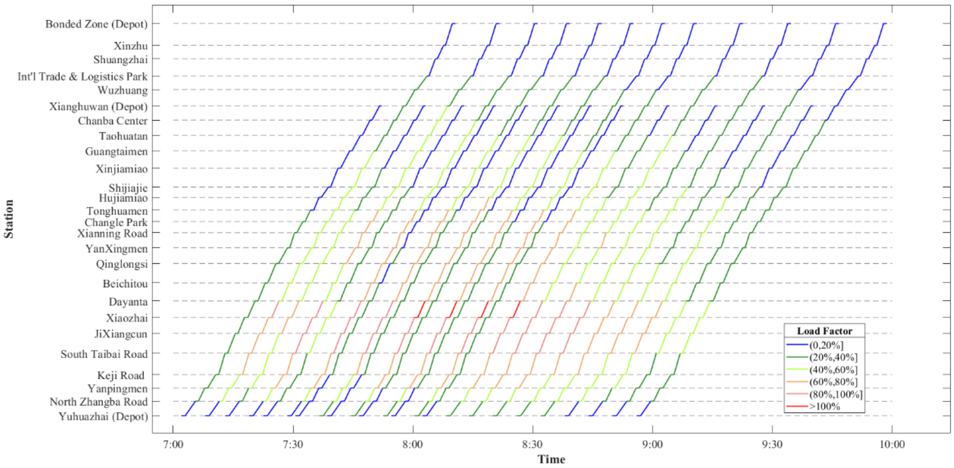
Table 6 Comparison of indicators under different passenger demand scenarios in Experiment 2

Optimal timetable	Scenario 6	Scenario 7	Scenario 8	Scenario 9	Scenario 10	Scenario 11	Average objective
$\boldsymbol{S_6}$	183.01	180.81	178.64	214.10	211.86	209.71	196.36
$\boldsymbol{S_7}$	183.02	180.81	178.63	214.00	211.74	209.55	196.29
$\boldsymbol{S_8}$	183.03	180.81	178.62	214.01	211.75	209.56	196.30
$\boldsymbol{S_9}$	186.01	183.75	181.53	206.40	206.40	201.85	194.32
$\boldsymbol{S_{10}}$	186.03	183.78	181.55	206.40	204.10	201.86	193.95
$\boldsymbol{S_{11}}$	186.06	183.78	181.54	206.43	204.11	201.83	193.96
$\boldsymbol{S_{robust2}}$	184.98	182.74	180.56	206.76	204.50	202.27	193.64

Then, we analyze the influence of passenger vehicle selection for the objective value. As shown in Table 6, for each optimal timetable, the objective value is smaller when the proportion of Type III in Type II is larger. For instance, the proportion of Type III in Type II are 0.1, 0.2, and 0.3 in Scenario 6, 7, and 8, respectively, and the objective values of each optimal timetable gradually decrease in these three scenarios. Furthermore, $S_{robust2}$ is also implemented in Scenario 5 of G_1, which doesn't consider passenger vehicle selection, and the train load factors in the upward direction are shown in Fig.8(a). The train load factors in the upward direction of $S_{robust2}$ implemented in Scenario 11 are displayed in Fig.8(b).



(a) Scenario 5 (not consider passenger vehicle selection)



(b) Scenario 11 (Consider passenger vehicle selection)

Fig. 8 Comparison of train load factor in the upward direction

The train load factors for the upward direction are similar in these two scenarios. In Scenario 5, the load factors of short-turning train services are much lower than full-length train services, because short-turning train services do not pass through all stations along this line, and can't carry Type II. However, in Scenario 11, 30% of Type II will take the short-turning train services to Xianghuwan Station, and transfer to the full-length train services. This unique vehicle selection behavior will transform some passengers from the full-length train services to the short-turning train services, and reduce the load factor variation between them, so the objective value in Scenario 11(202.27) is lower than Scenario 5 (209.28).

5 Conclusion

In this study, we proposed a robust optimization approach for train timetables and rolling stock circulation with short-turning strategy that involve uncertain passenger demand and vehicle selection. A robust optimization model was established by reducing the load factor variation as the optimization objective. In this model, we constructed several passenger scenarios to capture the uncertainty feature of passenger demand and vehicle selection. Furthermore, we divided the passengers into three types, and used the proportion of Type III (passengers who take the short-turning train services to the last station of the short-turning region, and transfer to the full-length train services to their destination stations) to represent the vehicle selection behavior in the short-turning region.

The Xi'an Metro Line 3 in China was adopted as a case study to test the effectiveness of the proposed method. Our results confirmed that the robust train timetable with short-turning strategy outperformed all full-length services timetables concerning the objective values and the rolling stock number. The robust train timetable can more effectively satisfy multi-scenario passenger demand than the satisfactory train timetable generated by independent optimization of each demand scenario, since if a train timetable is selected among these satisfactory timetables and implemented in all demand scenarios, the average objective value will be larger than for the robust train timetable. It can effectively improve the coupling degree between passengers and train services by reducing train capacity oversaturation and waste, and decrease the number of stranded passengers. In addition, we found the vehicle selection behavior (take the short-turning train services and transfer to the full-length train services) will transform some passengers from the full-length train services to the short-turning train services, and reduce the load factor variation between them.

Appendix A

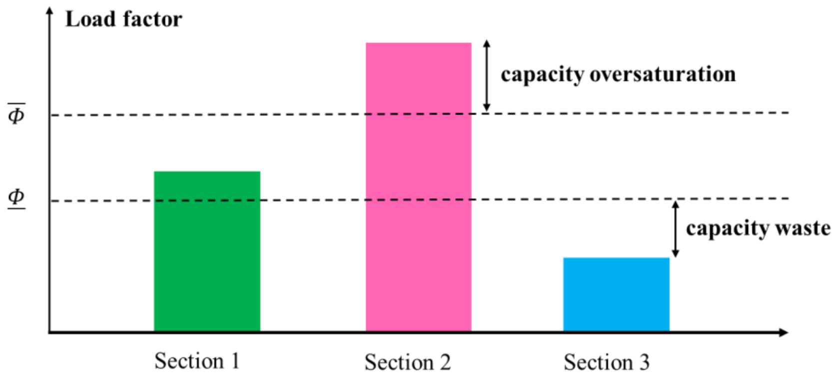


Fig A.1 A bidirectional train line with multi deports

We define two variables, $\overline{\Phi}$ and $\underline{\Phi}$, which refer to the upper and lower bounds of the "comfortable" load factor, respectively. $\partial 1^{x}_{r,i}(\psi,\nu)$ and $\partial 2^{x}_{r,i}(\psi,\nu)$ are the load factor variation values, which refer to the degree of train capacity oversaturation and waste, respectively. if $\theta^{x}_{r,i}(\psi)$ is within $[\underline{\Phi},\overline{\Phi}]$, $\partial 1^{x}_{r,i}(\psi)=0$, $\partial 2^{x}_{r,i}(\psi)=0$; if $\theta^{x}_{r,i}(\psi)$ is above $\overline{\Phi}$, $\partial 1^{x}_{r,i}(\psi)>0$, $\partial 2^{x}_{r,i}(\psi)=0$; if $\theta^{x}_{r,i}(\psi)$ is less than $\underline{\Phi}$, $\partial 1^{x}_{r,i}(\psi)=0$, $\partial 2^{x}_{r,i}(\psi)>0$.

Appendix B

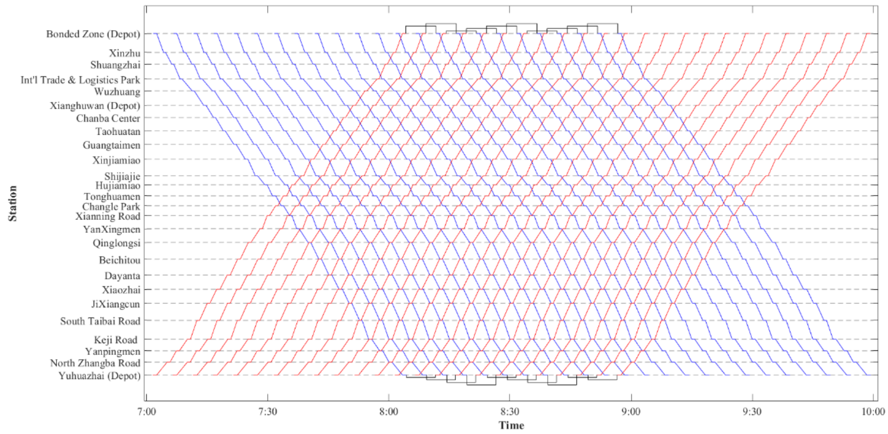


Fig A.1 Case I in Experiment 1

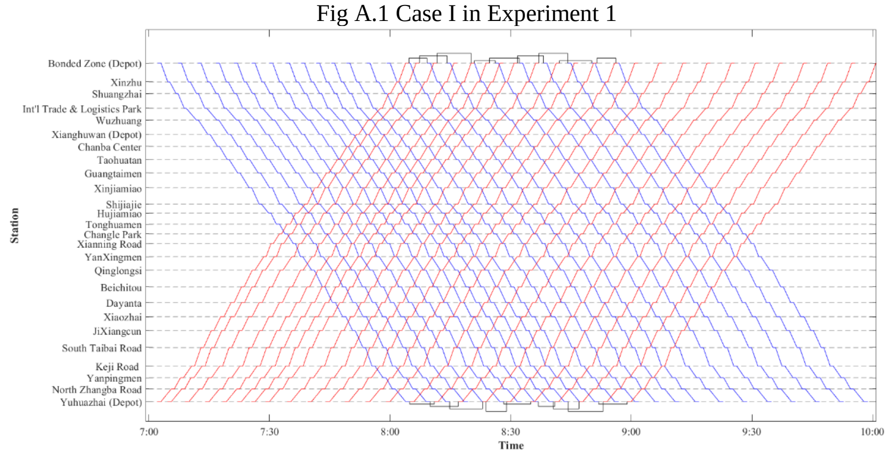


Fig A.2 Case II in Experiment 1

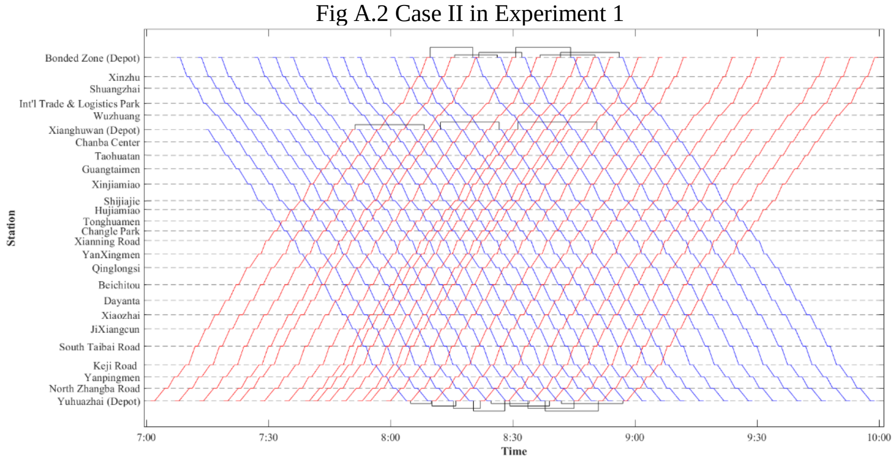


Fig A.3 Case III in Experiment 1

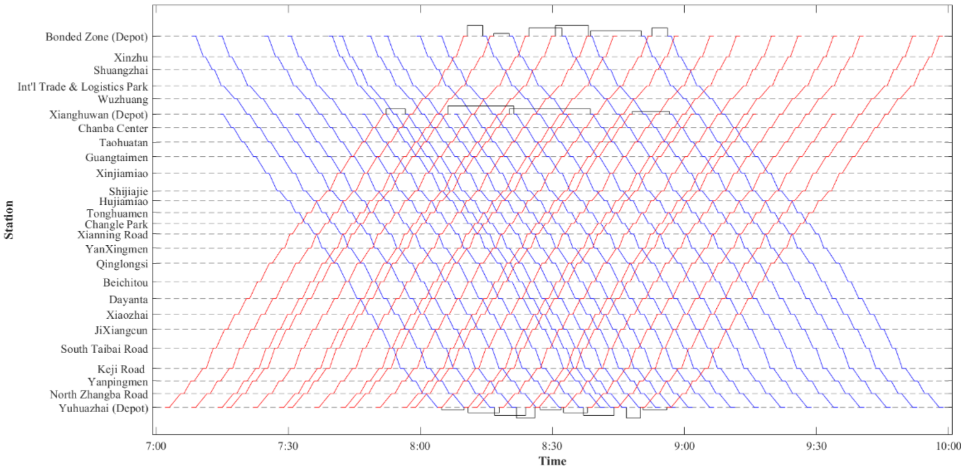


Fig A.4 Case VI in Experiment 1

Appendix C

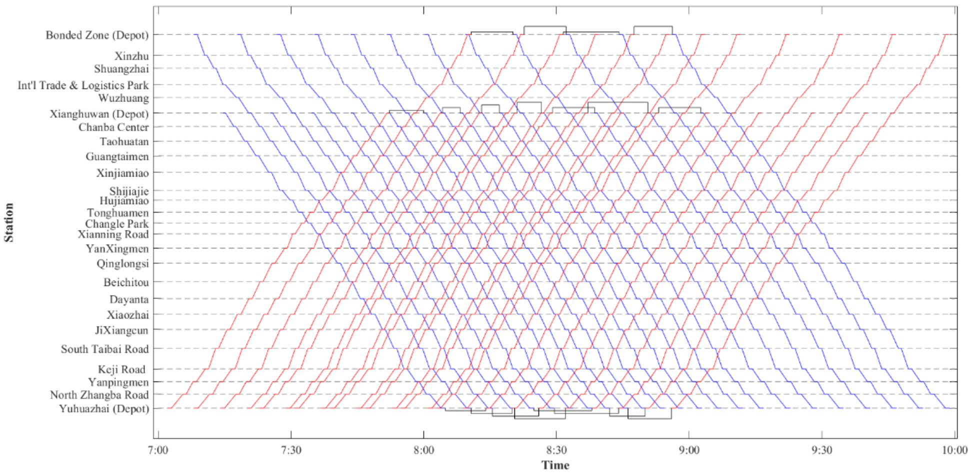


Fig B.1 Case V in Experiment 2 (Independent-mode).

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