# Paraboctys (part 1)

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# Paraboctys (part 1)

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**Abstract:** This study introduces the Paraboctys - Sieve of Primes, Quadratic Sequences of Primes, and Divisors.
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**2010 Mathematics Subject Classification:** 11N35, 11N36, 11A05, 11A51.

## 1 Introduction

A feeling of powerlessness arises when a subject deals with Prime numbers.

Every time one has tried to explain or define a Primo number throughout history, they have attempted to define it more clearly.

This was a consequence of the lack of a conclusive and universal equation to determine the Primo number sequences.

Controversy has existed over the number One being or not being a Prime number.

On May/16/2017, at https://en.wikipedia.org/wiki/Prime_number, the English definition stated: "*A Prime number (or a Prime) is a Natural number greater than 1 that has no positive divisors other than 1 and itself. A Natural number greater than 1 that is not a Prime number is called a Composite number.*"

In Brazilian Portuguese, the wording of the Prime number definition is so different that it seems the behavior of Primes would change according to the language in use.

On the same day, at https://pt.wikipedia.org/wiki/Número_primo, the Portuguese definition stated: *"A Prime number is a number P whose set of non-invertible divisors is not empty, and all its elements are products of P by invertible Integers. According to this definition, 0, 1 and -1 are not Prime numbers." ("Número primo, é um número P cujo conjunto dos divisores não inversíveis não é vazio, e todos os seus elementos são produtos de P por números inteiros inversíveis. De acordo com esta definição, 0, 1 e -1 não são números primos.").*

In contrast, on the same day, at https://es.wikipedia.org/wiki/Número_primo, the Spanish definition of Primes was an almost literal translation of the English version. It also adds that Composite numbers can be factored and the number 1 is neither a Prime nor a Composite by convention *("En matemáticas, un número primo es un número natural mayor que 1 que tiene únicamente dos divisores distinctos: él mismo y el 1. Por el contrario, los números compuestos* *son los números naturales que tienen algún divisor natural aparte de sí mismos y del 1 y por lo tanto, pueden factorizarse. El número 1, por convenio, no se considera ni primo ni compuesto."*).

To those who may be interested in this question, we suggest the analysis of several definitions of the Prime number put forward in all languages spoken throughout the world and history. Comparing the different translations of Pythagoras' theorem, for instance, maybe funny.

The definitions of a Prime number state either the properties they lack or those, which they have when we verify two results. Since in all cases we use the word "and" ("1 and itself") it is not possible to detect a prime directly in one-step.

The simplest and most objective definition of a Prime number I read was in *The Prime Facts: From Euclid to AKS © 2003 Scott Aaronson: "So what's a prime number? A whole number with exactly two divisors."*

## 1.1 Previous conventions:

Because our tables will show vertical sequences where the indexes will be on vertical and because on vertical, we have Y-axis in the XY-plane, so the sequences integers elements have to appear in X-axis as a function of the Y-axis.

Because of that, in all these studies we will represent any polynomial equation as being in the function of $y$, or just the function $Y[y]$, or $x = Y[y]$.

## 1.2 Notation for Polynomials In these studies

Generically we will denote any polynomial element as being $Y[y]$. When we want to draw the polynomial in the XY-plane, we will make $x$ in the function of $y$. In the Cartesian plane (square lattice grid) we can consider $x = Y[y]$. In different grid other than Cartesian plane $x \neq Y[y]$.

When we want to distinguish the $d^{\text{th}}$-degree of the polynomial, we will note $Yd[y]$ or $x = Yd[y]$.

When we want to make a $p^{\text{th}}$-power operation on an $d^{\text{th}}$-degree polynomial, we will note: $(Yd[y])^p$.

- Constant (polynomial degree 0) will be written as

$$Y0[y] = c$$

  One element determines this polynomial. We will express this as

$$Y0[y] \equiv [x_1]$$

- Linear (polynomial $1^{\text{st}}$-degree) will be written as

$$Y1[y] = by + c$$

  Two elements determine this polynomial. We will express this as

$$Y1[y] \equiv [x_1, x_2]$$

- Quadratic (polynomial $2^{\text{nd}}$-degree) will be written as

$$Y2[y] = ay^2 + by + c$$

Three elements determine this polynomial. We will express this as

$$Y2[y] \equiv [x_1, x_2, x_3]$$

- Cubic (polynomial 3rd-degree) will be written as

$$Y3[y] = a_3y^3 + ay^2 + by + c$$

Four elements determine this polynomial. We will express this as

$$Y3[y] \equiv [x_1, x_2, x_3, x_4]$$

- Quartic (polynomial 4th-degree) will be written as

$$Y4[y] = a_4y^4 + a_3y^3 + ay^2 + by + c$$

Five elements determine this polynomial. We will express this as

$$Y4[y] \equiv [x_1, x_2, x_3, x_4, x_5]$$

- Quintic (polynomial 5th-degree) will be written as

$$Y5[y] = a_5y^5 + a_4y^4 + a_3y^3 + ay^2 + by + c$$

Six elements determine this polynomial. We will express this as

$$Y5[y] \equiv [x_1, x_2, x_3, x_4, x_5, x_6]$$

And so on for Sextic, Septic, Octic, Nonic, Decic, etc.
Generic equation of polynomial $d^{\text{th}}$-degree:

$$Yd[y] = a_dy^d + a_{d-1}y^{d-1} + \cdots + a_4y^4 + a_3y^3 + ay^2 + by + c$$

Generically, we will adopt these equalities notation:

$$Yd[-3] = e$$
$$Yd[-2] = f$$
$$Yd[-1] = g = x_1$$
$$Yd[0] = h = x_2$$
$$Yd[1] = i = x_3$$
$$Yd[2] = j$$
$$Yd[3] = k$$

## 1.3 Notation for index direction in any polynomial sequence

Any polynomial Integer sequence has 2 directions. This is the reason any polynomial has 2 recurrence equations. So, if the direction is

$$Yd[y] \equiv (\dots, e, f, g, h, i, j, k, \dots) = \backslash(\dots, k, j, i, h, g, f, e, \dots)\backslash$$

then, the reverse direction is

$$\backslash Yd[y] \backslash \equiv (\dots, k, j, i, h, g, f, e, \dots) = \backslash(\dots, e, f, g, h, i, j, k, \dots)\backslash$$

## 1.4 Inflection point vs. vertex nomenclature

Because of the definition of the inflection point is in differential calculus *"an inflection point, point of inflection, flex, or inflection (British English: inflexion[citation needed]) is a point on a* *continuous plane curve at which the curve changes from being concave (concave downward) to convex (concave upward), or vice versa,*".

Because of the definition of the vertex: "*In geometry, a vertex (plural: vertices or vertexes) is a point where two or more curves, lines, or edges meet. As a consequence of this definition, the point where two lines meet to form an angle and the corners of polygons and polyhedra are vertices,*".

Because "*In the geometry of planar curves, a vertex is a point of where the first derivative of curvature is zero,*"

And like all studies between polynomials, no feature or phenomenon shows that there is a difference in behavior between quadratic and other polynomial orders, then there is no reason to differentiate the inflection point phenomena in quadratics from other polynomials. So, there is no reason to have different names.

In these studies, we will refer to this phenomenon in our tables, text, and figures as being only inflection points, even in quadratics which usually has the usual vertex name. The polynomials of a greater degree than quadratics will have two or more turning points besides the inflection point. But the common phenomenon among all polynomials is the inflection point.

The definition of a single Inflection Point nomenclature in common to all polynomials becomes important when we compare the behavior of the offset at all degrees.

In these studies, the coordinates of an inflection point in XY-plane are $x_{ip}$ and $y_{ip}$. Also, we will denote an inflection point as being $ip(x_{ip}, y_{ip})$.

## 1.5 Map of colors for all figures and tables

| Map of colors: |
|---|
| A000004 The Zero number, in red web color #FF0000. |
| A000012 The One number in blue light web color #3399CC. |
| A000040 The Prime numbers, in blue web color #336699. |
| A000290 The Square numbers (except Zero and One), in yellow web color #FFFF00. |
| A002378 The Oblong numbers (except Zero and Two), in red-dark web color #993333. |
| A005563 The Square minus One numbers (except 0 and -1), in Orange-dark web color #FF6600. |

# 2 Reflecting on the numbers Zero and One

The numbers Zero and One confuse us because they share several properties between the Primes and the Composites.

The first consequence of the need for verification of "1 and itself" brings to the conclusion that Zero and One cannot be Prime. If they were Prime, then the verification of Zero and One would consist of two steps, as it occurs in the Prime definition.

Also note that, differently from Primes, both numbers Zero and One are the only two Integers whose power of them, whatever it may be, results, respectively, in Zero and One.

$$0^n = 0$$
$$1^n = 1$$

Zero and One are the only Integers that cannot generate Composite numbers like any Prime can when multiplying by itself.

Since we do not know conclusively what happens when Zero divides any number, then we have to analyze the division of any number by One.

We cannot consider the number One a pure Prime because it has the following properties among others:

- It is a Square number, a Cube number, a Perfect n-power number. Therefore, the Unit is also a Composite number;
- The number One is the $n^{th}$ root of itself $\sqrt[n]{1} = 1$;
- The number One is the only number that inverts all numbers in $\mathbb{C}$;
- There is no other inverter of One than itself. The number One is the only number that inverts itself;
- Repunits are generated only by numbers One's;
- No other Integer would fit to include only itself indefinitely in the continued fraction (see Equation of A165900 in offset zero);

Several properties apply only to the Integers Zero and One.

The lack of knowledge of a conclusive mathematical structure that defines the random/predictable appearance of Primes gives rise to many conjectures that we need to prove.

In contrast, several sieves (filters) show us the behavior of the "sequencing" of Primes.

We show some sieves below:

- Sieve of Eratosthenes,
- Divisor Drips and Square Root Waves, Box Factor
- Otto Belden blog
- Quadratic Sieve
- Ulam's spiral
- NumberSpiral
- …

However, it is the same polynomial principle that explains all these sieves.

This is because all the above sieves obey the method of finite differences.

The method of finite differences explains all polynomials.

It is the method of finite differences that explains why the derivative or integral polynomial results in another polynomial of a lesser or greater degree.

See chapter Polynomial Properties for more details.

# 3 Reflecting on the numbers Zero, One, and Primes

The characteristic property of Prime numbers is that we cannot factorize them. The Prime number cannot be Composite, the product of two or more Natural numbers different from Zero or One. This property explains what a Prime number is not.

The first intriguing issue emerges: are Zero and One Prime numbers, or not?

Zero and One fit into the characteristic property that they cannot result from the product of two other Natural numbers, since each of them is always the result of a product that includes itself.

With the number Zero, there are infinitely many products of two Natural numbers which result in Zero, where at least one factor is Zero in all cases.

With the number one, "two other Integers" are both equal to each other.

Notice that these characteristic properties of Zero and One are unique and do not apply to any Prime or Composite number.

Based upon the characteristic property of numbers Zero and One, if **Prime numbers are <u>all</u> the numbers that result from only <u>one</u> product of Natural numbers, with only <u>one</u> factor of that product being equal to number <u>One</u>**, then the numbers Zero and One are not Primes and we cannot factorize the Prime numbers.

Because Prime numbers result from only one product of Natural numbers, this does not allow the number Zero to be a Prime number.

Because only one factor of that product is equal to One, this does not allow the number One to be a Prime number.

Although the number One is not a Prime number, it is the only Integer number mentioned explicitly 3 times to define what Prime numbers are. We can add a fourth use of the number One when we use the word "all". Here, we understand "all" as "infinite", represented by 1 inverting 0 (or 0 dividing 1). Thus, Zero and One are not Primes, but only the numbers Zero and One are used to defining Prime numbers.

Allowing the mind to flow, before Zero divides all the Natural numbers 1, 2, 3, 4,... to become a common undetermined result, we conjecture that Zero divides by itself generating the number One.

## 3.1 Prime 2

Many are the consequences stemming from this characteristic property that shows what a Prime is not. As for the decimal base, the first clear finding is that a Prime number cannot be a multiple of 2.

It is usually said that '*all Prime numbers are Odd numbers. A Prime number always results from the sum described as (Even+1) = (2n+1),* $n \in N$ *or Prime Number* $\neq n \pmod{10} \in \{0, 2, 4, 6, 8\}$'. However, since 2 is not an Odd number and $2 \pmod{10} \in \{0, 2, 4, 6, 8\}$, then the second intriguing issue of the journey emerges: is the number 2 a Prime or not?

According to the definition stating that **Prime numbers are all the numbers that result from only one product of Natural numbers, with only one factor of that product being equal to number One**, then number 2 is a Prime number, even if it is the oddest Even Prime number.

Prime 2 is the lowest prime number. Any prime number is a divisor in an infinite number of composite numbers. It will always have its lowest Composite value when multiplied just by the Prime 2.

## 3.2 Prime 5

Since we carry this work out in the decimal base, we must point to the function of 2 in the decimal base. In the decimal base, 2 plays an important role to be one of the two factors of 10. The other factor is number 5. So, the second consequence is that the Prime number in the decimal base cannot be a multiple of 5. It is usually said that '*all Prime numbers are Odd numbers that never have unit digit the numbers 0 or 5. Alternatively, Prime number ≠ n (mod 10) ∈ {0, 5}*'. The third intriguing issue is whether 5 is a Prime number.

By definition, **Prime numbers are all the numbers that result from only one product of Natural numbers, with only one factor of that product being equal to number One**, the number 5 is thus a Prime, even though it is the oddest Odd Prime number that ends in 5.

Also, it is important to highlight that, in base 10 (and 10=2*5), modular arithmetic (mod 10) determine any Composite multiple of 2 and/or 5:

$Prime\ Number \neq n\ mod\ 10\ \in\ \{0,2,4,5,6,8\}$, except Primes 2 and 5.

Modular operations $n\ mod\ 10$ will identify all Composites multiple of 2 and/or 5 by taking into account just the unit digit.

## 3.3 Prime 3

The digit set of the decimal base comprises {0,1,2,3,4,5,6,7,8,9}. If we factorize all digits, we get the following data: $\{0, 1, 2, 3, (2*2), 5, (2*3), 7, (2*2*2), (3*3)\}$. So, the set of primary factors is $\{0, 1, 2, 3, 5, 7\}$. According to the definition that **Prime numbers are all the numbers that result from only one product of Natural numbers, with only one factor of that product being equal to number One**, the Integers Zero and One are not Primes. The conclusion that follows is that the Prime numbers among the primary factors of base 10 are the set {2, 3, 5, 7}. Remain to analyze deeper the digits 3 and 7.

It is usually said that '*no multiple of 3 can be Prime, no Prime number can have digital root (repeated digit sum, or casting out nines) with values {3, 6, 9} or Prime number ≠ n mod 9 ∈ {0, 3, 6}*'. The fourth intriguing issue is whether 3 is a Prime number. By definition **Primes are all numbers that result from only one product of Natural numbers, with only one factor of that product being equal to number One**, 3 is a Prime number, even though it is the oddest Odd Prime number with digit sum of value 3.

## 3.4 Prime 7

The adopted definition that **Prime numbers are all the numbers that result from only one product of Natural numbers, with only one factor of that product being equal to the number One** also applies to 7.

It would seem that 7 is the first Prime, number not associated with any intriguing issue. However, Integer 7 brings a powerful message on Prime numbers.

Every time we perform an Integer number factorization, we use the division operation.

Every time we use the division operation we use the concept of the reciprocal.

At base 10, it is usually said that '*Primes 2 or 5 are invertible without repeating decimal digits*', whereas reciprocals of 3 or 7 result in repeating decimal digits.

- $\frac{1}{1} = 1$ and $\frac{10}{1} = 10$ because $\frac{2*5}{1} = 10.\overline{0}$;
- ½ =0.5 and $\frac{10}{2} = 5$ because $\frac{2*5}{2} = 5.\overline{0}$;
- $\frac{1}{5} = 0.2$ and $\frac{10}{5} = 2$ because $\frac{2*5}{5} = 2.\overline{0}$;

See the properties of the repeating decimal digits of the reciprocals of 3 and 7 below:

- $\frac{1}{3} = 0.\overline{3}$ and $\frac{10}{3} = \frac{2*5}{3} = 3.\overline{3}$ because (2, 5, 3) are coprime.
- $\frac{1}{7} = 0.\overline{142857}$ and $\frac{1000000}{7} = \frac{2^6*5^6}{7} = 142857.\overline{142857}$ because (2, 5, 7) are coprime.

In conclusion, Prime 7 is the lowest Prime where its reciprocal generates a repeating digit different from $\overline{0}$.

Because of the Repetend Theorem, all reciprocal generates a repeating digit $mod\ 9 = 0$.

### 3.4.1 Repetend Theorem

Let's work on the base10.

Be the number prime *P* different of 2 and 5 and its reversal given by $\frac{1}{P}$.

$$\frac{1}{P} = a_0, \overline{a_1 a_2 a_3 a_4 \dots a_n}$$

$$\frac{1}{P} = \frac{a_1 a_2 a_3 a_4 \dots a_n}{9999 \dots 9}$$

Where the repeating decimal (repetend) is composed of the elements $\overline{a_1 a_2 a_3 a_4 \dots a_n}$ with period n. So,

$$\frac{1}{P} = q + \frac{r}{P}$$

In the first division iteration, we get

$$\frac{1}{P} = 0 + \frac{1}{P}$$

Note that 0 is the $q$ quotient of the $\frac{1}{P}$ division. Then, it is the first digit $a_0$ before the decimal separator, and 1 is the first remainder $r_0$ from $\frac{1}{P}$.

Because of the prime P>5, then always

$$a_0 = 0$$
$$r_0 = 1$$

Now

$$\frac{10 * r_0}{P} = \frac{10 * 1}{P} = \frac{10}{P} = \mathrm{a}_1. a_2 a_3 a_4 \dots a_n a_1 a_2 a_3 a_4 \dots a_n \dots$$

$\frac{10*r_0}{P} = \frac{10}{P} = \mathrm{a}_1 + \frac{r_1}{P}$, where $a_1$ is the quotient, $r_1$ is the remainder from $\frac{10}{P}$

$\frac{10*r_1}{P} = \mathrm{a}_2 + \frac{r_2}{P}$, where $a_2$ is the quotient, $r_2$ is the remainder from $\frac{10*\mathrm{r}_1}{P}$

$\frac{10*r_2}{P} = \mathrm{a}_3 + \frac{r_3}{P}$, where $a_3$ is the quotient, $r_3$ is the remainder from $\frac{10*\mathrm{r}_2}{P}$

...

$\frac{10*r_{n-1}}{P} = a_n + \frac{r_n}{P}$, where $a_n$ is the quotient, $r_n$ is the remainder from $\frac{10*r_{n-1}}{P}$

See that $r_n = 1$, otherwise repeating decimal (repetend) would not have period n.

Also note that the remainders $r_n$ are nothing more than the value of the power of 10 mod P. That is, to calculate the repeating decimal (repetend) digits of a prime number reversal, it is also calculated $10^n = r_n \ (mod\ P)$.

For $10^n = 1 \ (mod\ P)$ means that repeating decimal (repetend) has come to an end and it will start over.

Therefore, to calculate the value of the Digital Root (DR) of a repeating decimal (repetend) produced by $\frac{1}{P}$:

$$DR(a_0, a_1 a_2 a_3 \dots a_n) = DR(a_0 + a_1 + a_2 + a_3 + \cdots + a_n)$$

$$a_0 = 0$$

$$a_1 = \frac{10 * r_0}{P} - \frac{r_1}{P} = \frac{10}{P} - \frac{r_1}{P}$$

$$a_2 = \frac{10 * r_1}{P} - \frac{r_2}{P}$$

$$a_3 = \frac{10 * r_2}{P} - \frac{r_3}{P}$$

...

$$a_n = \frac{10 * r_{n-1}}{P} - \frac{r_n}{P} = \frac{10 * r_{n-1}}{P} - \frac{1}{P}$$

$$DR(a_0, a_1 a_2 a_3 \dots a_n) = DR\left(0 + \frac{10}{P} - \frac{r_1}{P} + \frac{10 * r_1}{P} - \frac{r_2}{P} + \frac{10 * r_2}{P} - \frac{r_3}{P} + \cdots + \frac{10 * r_{n-1}}{P} - \frac{1}{P}\right)$$

$$DR(a_0, a_1 a_2 a_3 \dots a_n) = DR\left(\frac{10 - r_1 + 10r_1 - r_2 + 10r_2 - r_3 + 10r_3 - \cdots - r_{n-1} + 10r_{n-1} - 1}{P}\right)$$

$$DR(a_0, a_1 a_2 a_3 \dots a_n) = DR\left(\frac{9 + 9r_1 + 9r_2 + 9r_3 + \cdots + 9r_{n-1}}{P}\right)$$

Because no Prime is divisible by 3, then $DR(P) \neq 3,6,9$. So,

$$DR(a_0, a_1 a_2 a_3 \dots a_n) = DR\left(9 * DR\left(\frac{1 + r_1 + r_2 + r_3 + \cdots + r_{n-1}}{P}\right)\right)$$

$$DR(a_0, a_1 a_2 a_3 \dots a_n) = DR(9)$$

$$DR(a_0, a_1 a_2 a_3 \dots a_n) = 9$$

In conclusion, the Digital Root of all the repeating decimal (repetend) generated by the reversal of prime numbers is always 9.

Remember that

$$\frac{1}{P} = \frac{a_1 a_2 a_3 a_4 \dots a_n}{9999 \dots 9}$$

So,

$$DR\left(\frac{1}{P}\right) = DR\left(\frac{a_1 a_2 a_3 a_4 \dots a_n}{9999 \dots 9}\right)$$

As

$$DR(a_0, a_1 a_2 a_3 \dots a_n) = 9$$

So, the $DR\left(\frac{1}{P}\right)$ will depend on n. For each n value, there will be a different factoring of the denominator

$$10^{(n-1)} - 1 = 999 \dots 9$$

For example,

$$DR\left(\frac{1}{7}\right) = DR\left(\frac{142857}{999999}\right)$$

$$DR\left(\frac{1}{7}\right) = DR\left(\frac{3^\wedge 3 * 11 * 13 * 37}{3^\wedge 3 * 7 * 11 * 13 * 37}\right)$$

Let's eliminate the threes. In the field of Digital Root arithmetic, 3 in denominator destabilizes our mathematics, in the same way as division by zero.

Thus, the first thing to do is to eliminate the 3's in common between the numerator and denominator:

$$DR\left(\frac{1}{7}\right) = DR\left(\frac{11 * 13 * 37}{7 * 11 * 13 * 37}\right)$$

$$DR\left(\frac{1}{7}\right) = DR\left(\frac{11 * 13 * 37}{7 * 2 * 4 * 1}\right)$$

$$DR\left(\frac{1}{7}\right) = DR\left(\frac{5291}{56}\right)$$

$$DR\left(\frac{1}{7}\right) = DR\left(\frac{8}{2}\right) = 4$$

That is, to calculate the value of the Digital Root of the Prime Numbers reversal, we have to factorize the repeating decimal (repetend) and divide it by the $(10^{(n-1)} - 1)$ factorized.

## 3.5 Prime digits summary

A summary of the study findings is shown below:

- Number 0 is not Prime, because there are infinitely many Integer products that result in Zero.
- Number 1 is not Prime, because the only product that results in One comprises both factors of the same value.
- Number 2 is Prime and gives rise to the decimal digits {2, 4, 6, 8}, and both 2 and 5 give rise to the decimal digit {0}. Modular arithmetic (mod 10) detects multiples of 2.
- Number 3 is Prime and gives rise to the decimal digits {3, 6, 9}. Modular arithmetic $mod\ 9 = (3,6,9)$ detects multiples of 3.
- Number 4 is not Prime, because it is multiple of Prime 2. It is the lowest Composite.
- Number 5 is Prime and gives rise to the digit {5}, and both 2 and 5 give rise to the decimal digit {0}. Modular arithmetic (mod 10) detects multiples of 5.
- Number 6 is not Prime, because it is multiple of 2. Notice the if we consider only the lower Prime factor, then we can reduce any Composite to only 2 terms product.
- Number 7 is Prime and gives rise to the decimal digit {7}. Modular arithmetic (mod 7) detects multiples of 7.
- Number 8 is not Prime, because it is a multiple of 2.
- Number 9 is not Prime, because it is a multiple of 3.

It is worth a note about the first 4 Prime numbers:

- Number 2 is the first Prime number of all Prime numbers.

- Number 3 is the first Prime number of all Odd Prime numbers.
- Number 5 is the first Prime number after the first Composite number.
- The number 7 is the first Prime number in which its reciprocal results in a periodic repetend such as $(repetend)\ mod\ 9 = 0$. Or the $Digital\ Root\ (repetend) = 9$. All Prime numbers greater than 7 will have the reciprocal generating repetend in which $(repetend)\ mod\ 9 = 0$. Or the $Digital\ Root\ (repetend) = 9$.

## 3.6 Intriguing issues?

These findings show us that Primes 2, 3, 5 are as odd as Prime 7. Also, the intriguing issues related to 2, 3, 5 are not, in fact, "intriguing issues". The "intriguing issue" related to 2, 3, 5, described earlier, also applies to all Primes following them, as shown for the Prime 7.

The "intriguing issue" is the intrinsic property: **Prime numbers are all the numbers that result from only one product of Natural numbers, with only one factor of that product being equal to number One**.

Paulo Ribenboim writes in The Little Book of Bigger Primes on page 2: "*Which is the oddest Prime number? It is 2 because it is the only Even Prime number!"*. Inspired by his intriguing question, a subject can indefinitely repeat that same question and give infinite different answers, one for each Prime number we can know:

- Which is the oddest Prime number? It is 3 because it is the only triple Prime number!
- Which is the oddest Prime number? It is 5 because it is the only quintuple Prime number!
- Which is the oddest Prime number? It is 7 because it is the only septuple Prime number!
- Which is the oddest Prime number? It is 11 because it is the only undecuple Prime number!
- Which is the oddest Prime number? It is 13 because it is the only tredecuple Prime number!
- ...
- Which is the oddest Prime number? It is Prime because it is the only Prime-tuple Prime number!

And so on.

All Primes have the same intrinsic equivalent of oddest property, with no exception to the rule: **Prime numbers are all the numbers that result from only one product of Natural numbers, with only one factor of that product being equal to number One**.

## 3.7 Modular arithmetic

When we use (mod 10), we use simultaneously (mod 2) and (mod 5) because (mod 10) detects multiples of both 2 and 5. (mod 9) detects multiples of 3, 6, and 9 at the same time.

When the modular arithmetic (mod 7) is used to analyze all the Natural numbers already analyzed with (mod 10) and (mod 9), then we show that the only Prime number divisible by 7 is the number 7.

This is an intriguing question not seen before because we can only verify it when we apply (mod 7).

The 10 decimal digits set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} contain Composites made only by Primes factors 2, 3, and 5. The smallest Composites are {2*2, 2*3, 2*2*2, 3*3}, all of them contained within the 10 decimal digits {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

Prime 7, (mod 7) identifies the number 14 as the smallest multiple composed by 7. The number 14 does not belong to the 10 decimal digits {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. Thus, only (mod 9) and (mod 10) can detect the composites of the 10 decimal digits {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

Based on previous questions and answers, the following conclusions result:

- Modular Arithmetic detects all Composite numbers.
- Modular Arithmetic is useful to explain why 0 and 1 cannot be Primes: there is no (mod 0) or (mod 1). They do not apply to differentiate Primes from Composites.
- Because we are in base 10, then (mod 9) operations are equivalent to the Digital Root operation of any Integer. To get digital root value, we change $n \, mod \, 9 = 0$ by $n \, mod \, 9 = 9$. In fact: $0 \, mod \, 9 \equiv 9 \, mod \, 9$.
- Because we are in base 10, (mod 10) gives us the Unit digit of any Integer.
  - The (mod 9) looks for the "Zero" in each Integer. The (mod 10) looks for the "Unit" in each Integer.
  - The (mod 9) and (mod 10) have only a single-digit as results. They cover all base 10 digits.

## 3.8 The Game Played by Zero, One, Primes, and Composites

This is the great game Natural numbers play:

- Composites are always the result of the multiplication of two Integers different from 0 and 1.
  - Composite=N*M, where {N,M}≠{0,1}.
  - Composites are multiplications without Unit or Zero.
- Prime numbers are always the result of the addition of either two Units or a Unit plus an Even Natural number other than Zero.
  - Prime=1+1 or Prime≡(2n+1), n>0.
  - Primes are the addition of the Unit.
- The number Zero cannot be a composite number because there are no two Natural numbers, both different from Zero, which multiplied each other results in 0.
- The number Zero cannot be a Prime number because there are no two Natural numbers, both different from Zero, that added together result in 0.
- The number One cannot be a composite number because there are no two different Natural numbers which multiplied among themselves results in 1.
- The number One cannot be a prime number because there are not two Natural numbers, both different from Zero, or both different from One, which added together result in 1.

The above rules, used throughout these studies, give rise to unexpected discoveries.

## 3.9 Conclusions from Prime Numbers

When we analyze the sequence of Natural numbers, first we study numbers Zero and One. The conclusion is that both are not Primes, neither Composites.

Next, we analyze number 2 established as the first Prime, of the sequence of Natural numbers. Thus, we identify all composite numbers which are a multiple of 2. The number 4 is the first composite number because it is the product of the first Prime by itself: 4=2*2=2^2.

The Square number is the first Composite number generated by a newly identified Prime number without the presence of the Prime 2 as a factor. For example:

- The next number is Prime 3. The Prime 3, multiplied by itself, gives rise to composite 9. And the next composite is 6 because it is the product of primes 2 and 3.
- Primes 2 and 3 do not cover number 5. So, the next Prime is 5. The Prime 5 multiplied itself, gives rise to composite 25. Prime 2 generates numbers 10 and 20 and Prime 3 generates the number 15.
- and so on…

Consider that every Prime number P possesses an image of all Natural numbers greater than itself. The Prime number P immediately generates the infinite Composites multiple of P, thus giving rise to the following sequence of new Composites: $\{P * P, P * (P + 1), P * (P + 2), P * (P + 3), \ldots\}$.

As a result, we can foresee the generation of all Composite numbers that are multiples of P from P, the first of them is the Square number of P.

The finding of $8 = 2 * 4 = 4 * 2 = 2 * 2 * 2 = 2^3$ shows that, after finding the Square numbers and the product of the Prime number P by all Natural numbers greater than P, there are the perfect cube numbers.

The latter is followed by the Composite numbers generated by the Prime number P multiplied by two Primes that can be equal to each other or not. We present the example below for the first Prime number:

- 2
- 2*2
- 2*(Prime>2)
- 2*2*2
- 2*(Prime>2)*(Prime >2)
- 2*2*2*2
- 2*(Prime >2)*(Prime >2)*(Prime >2)

# 4. There is no last digit

Once multiplication and division can detect Prime numbers, it is important to analyze the reciprocal of the 10 decimal digits {1, 2, 3, 4, 5, 6, 7, 8, 9, 0} precisely.

The correct way to study the reciprocal of the 10 decimal digits is to consider that there is no difference between Primes 2, 3, 5, and 7 regarding an "intriguing question". The intrinsic properties of the Primes 2, 3, 5, and 7 have to be strictly the same, with no difference. The equality of properties also applies when we do multiplications or their reciprocal. How does it happen?

Both $\frac{1}{3}$ and $\frac{1}{7}$ suggest a good tip. Both reciprocals have visible repetends. This means that all other Primes have repetends. Reciprocals should be like:

$$\frac{1}{1000} = 0.001\overline{0}$$
$$\frac{1}{100} = 0.01\overline{0}$$
$$\frac{1}{10} = 0.1\overline{0}$$
$$\frac{1}{1} = 1.\overline{0}$$
$$\frac{1}{2} = 0.5\overline{0}$$
$$\frac{1}{3} = 0.\overline{3}$$
$$\frac{1}{4} = 0.25\overline{0}$$
$$\frac{1}{5} = 0.2\overline{0}$$
$$\frac{1}{6} = 0.1\overline{6}$$
$$\frac{1}{7} = 0.\overline{142857}$$
$$\frac{1}{8} = 0.125\overline{0}$$
$$\frac{1}{9} = 0.\overline{1}$$

and so on ...

The reciprocal of Prime numbers show us that any number has infinitely many digits (repeating digits).

The last digit does not exist.

Any reciprocal of an Integer has repetends.

Even a Natural or an Integer number does not have a “last” digit.

All of them have infinitely many decimal digits, like any sequence generated by a polynomial with infinite many indexes.

When the finite string of significant digits finishes an Integer number, the zero strings to the left and the right side of the decimal point will fill all the infinite many digit places.

Any number has a unit digit, a hundred digits, and so on up to infinity.

All Integers have decimal digit up to infinity.

Do not exist one last digit.

Any (Integer (mod 10)) cannot represent the “last” digit since it represents the unit digit.

This is very important in this work because the unit digits of the numbers form a sequence of aligned (synchronized) digits.

Decimal digits alignment and synchronization occur in the same way that the Integer elements of polynomial sequences occur.

After understanding what this work is, maybe a subject can consider the Unit digit to be the first digit (as a subject can consider the first Integer elements in a polynomial sequence), but never the last one!

We use this concept when we study the modular arithmetic table in polynomial sequences, repeated modularity strings, and repeated modularity value results.

It is a concept similar to the concept of repeating decimals.

# 5. Integer number

The Integer number results from the Floor function applied in infinitely many Real numbers.

$$Integer = \lfloor Real \rfloor$$

We can see the floor function as resetting any decimal digit after the decimal separator.

$$Integer = \lfloor Real \rfloor = (\ Integer\ digits\ )(decimal\ point)(\ \overline{0}\ )$$

Any Integer is the application of Zeros in all infinitely many decimal positions of infinitely many real numbers.

That’s why it is possible to exist a Prime number.

# 6. The form of all Primes Numbers

Because of the first Prime number 2, all Prime numbers are in the form (1+1) for Prime 2 and the form (2n+1) for all Primes > 2.

Because of the Prime number 3, all Prime numbers are in the form (1+1) for Prime 2, (2+1) for Prime 3 and the form ((2n+1) and (3n+-1)) for all Primes > 3. When the data are combined, all Prime numbers can be represented by the form (1+1) for Prime 2, (2+1) for Prime 3, and the form (6n+-1) for all Primes > 3.

Because of the Prime number 5, all Prime numbers are in the form (1+1) for Prime 2, (2+1) for Prime 3, (6-1) for Prime 5, and the form ((6n+-1) unit digit different from 5) for all Primes > 5.

Because of the Prime number 7, all Prime numbers are in the form (1+1) for Prime 2, (2+1) for Prime 3, (6-1) for Prime 5, (6+1) for Prime 7, and the form (((6n+-1) unit digit different from 5) not divisible by 7) for all Primes > 7.

Because of the Prime number 11, all Prime numbers are in the form (1+1) for Prime 2, (2+1) for Prime 3, (6-1) for Prime 5, (6+1) for Prime 7, (6*2-1) for Prime 11, and the form (((6n+-1) unit digit different from 5) not divisible by (7 nor 11)) for all Primes > 11.

Because of the Prime number 13, all Prime numbers are in the form (1+1) for Prime 2, (2+1) for Prime 3, (6-1) for Prime 5, (6+1) for Prime 7, (6*2-1) for Prime 11, (6*2+1) for Prime 13, and the form (((6n+-1) unit digit different from 5) not divisible by (7, 11 nor 13)) for all Primes > 13.

Because of the Prime number 17, all Prime numbers are in the form (1+1) for Prime 2, (2+1) for Prime 3, (6-1) for Prime 5, (6+1) for Prime 7, (6*2-1) for Prime 11, (6*2+1) for Prime 13, (6*3-1) for Prime 17 and the form (((6n+-1) unit digit different from 5) not divisible by (7, 11, 13 nor 17)) for all Primes > 17.

...

Because of the Prime number P, all Prime numbers are in the form (1+1) for Prime 2, (2+1) for Prime 3, (6-1) for Prime 5 and the form (((6n+-1) unit digit different from 5) not divisible by any (7 <= Prime <= P)) for all others Primes > P.

## 6.1 Factor properties in Primes Numbers

Integer 2 is the first Prime, which is not a factor of Unit's product $(1 * 1)$. Prime 2 is the first Prime of all Primes. All Primes bigger than 2 have this property.

Integer 3 is the first Prime, which is not a factor of Composites of the form $(2n)$. Prime 3 is the first Odd Prime. All Primes bigger than 3 have this property.

Integer 5 is the first Prime, which is not a factor of Composites of the form $(2n)$ and $(3n)$. All Primes bigger than 5 have this property.

Integer 7 is the first Prime, which is not a factor of Composites of the form $(2n)$ and $(3n)$ and $(5n)$. All Primes bigger than 7 have this property.

Integer 11 is the first Prime, which is not a factor of Composites of the form $(2n)$ and $(3n)$ and $(5n)$ and $(7n)$. All Primes bigger than 11 have this property.

Integer 13 is the first Prime, which is not a factor of Composites of the form $(2n)$ and $(3n)$ and $(5n)$ and $(7n)$ and $(11n)$. All Primes bigger than 13 have this property.

…

Integer P is the first Prime, which is not a factor of Composites of the form $(2n)$ and $(3n)$ and $(5n)$ and $(7n)$ and $(11n)$ and $((any\ prime < P) * n)$. All Primes bigger than P have this property.

Conclusion: there is not a single form for any two Prime numbers. We describe each Prime number differently from other Primes. Each Prime has its oddest characteristic that applies only to itself.

There is not any single form to describe two Prime numbers, whatever they may be.

This is a striking difference, as compared to the forms (or mathematical properties) describing the elements of a number sequence generated by a polynomial.

The form or the mathematical properties of a polynomial sequence element repeat periodically along with other elements.

Similarly, it occurs in repeating decimals of Prime reciprocals or exponential sequences.

But the form or mathematical properties of Primes never repeat themselves.

The form of repetition cycle and the mathematical properties of a polynomial sequence has a finite size like repeating decimals of Prime reciprocals.

The form of repetition cycle and the mathematical properties of the sequence of Prime numbers has an infinite size, as infinite as the number of elements in a polynomial sequence.

## 6.2 Conjecture

Considering that any polynomial sequence has infinitely many indexes and, therefore, infinitely many elements;

Considering that any mathematical operation will occur repeatedly all along with its polynomial elements; and

Considering the form of all Prime numbers never repeat.

So, it is not possible to find any polynomial with infinite many Primes elements in a sequence of Primes, but at most a finite sequence of Primes with a maximum size for that polynomial.

## 6.3 Conjecture

Considering that all prime numbers have the same basic characteristic of being the only divisors of themselves different from 1, then if a polynomial sequence can generate at least a few prime numbers, then it can generate infinitely many others.

This is because if a polynomial sequence that is irreducible and has no variable in common could only generate Composites, then there would be some other polynomial that would only generate prime numbers.

But since all polynomials obey the method of finite differences, then no polynomial can generate an infinite sequence of many prime numbers.

It is also an observation of the covering system.

### 6.4 Conjecture

Because the law of formation of Composites is straightforward without alternatives or variations, it is possible to determine one or more equations that generate all the Composite numbers of a polynomial sequence.

The Prime numbers will be all those that belong to the polynomial sequence, but not to the elements of the Composite generator equation. Since the law of formation of Primes is not straightforward and has alternatives or variations, it is not possible to determine an equation that generates all the Primes of a polynomial sequence.

### 6.5 Conjecture

Considering that any polynomial sequence has infinitely many indexes and, therefore, infinitely many elements, then all finite subsets of its elements will result from the equality between the polynomial sequence equation and another function. All finite sequences are thus of the form function 1 equal to function 2.

## 7. Reflecting on Prime Number Sequencing

Many are the equations and algorithms that try to generate a big sequence of Prime numbers. However, all of them have some sort of disadvantage and/or limitation.

Let's make a trial in the practice.

Let's start our reasoning by thinking all Prime numbers are of the form (6n±1), except Primes 2 and 3.

Because of this exception, we have already started imperfect.

But let's neglect because we have the feeling that this imperfection is "tiny".

They are only two Primes of infinitely many Prime numbers.

Since a Prime number cannot result from a multiplication of two integers different from the number One and the Prime itself, then the first idea that comes to mind is that no Square or Cube number, nor any other power can be Prime:

$$Prime \neq n^m, n \neq 1, m \neq 1$$

Broadly, a Prime cannot result from multiplying different Integers, i.e.:

$$Prime \neq n(n \pm m)$$

$$Prime \neq n(n \pm m)(n \pm l) \dots (n \pm o)$$

nor can it be a combination of all forms:

$$Prime \neq n^p (n \pm m)^i (n \pm l)^j \dots (n \pm o)^g$$

$$Prime \neq sn^p (n \pm m)^i (n \pm l)^j \dots (n \pm o)^g$$

The following equation presents a generic notation:

$$Prime \neq n(n \pm k), where\ n \neq k\ and\ (n, k) \in Z$$

Notice that we can factorize every Composite into only two factors since each factor can also be a Composite or a Prime and the difference between the two factors can be any Integer $k$.

Therefore, to denote what a Prime cannot be, the expression above $Prime \neq n(n \pm k)$ is fully complete and unique.

Notice that this also applies to $Prime \neq sn^p(n \pm m)^i(n \pm l)^j \dots (n \pm o)^g$, which we reduce to the multiplication of only two terms in all cases.

In conclusion, this is what a Prime number is not: $Prime \neq n(n \pm k)$.

## 7.1 First trial to find sequences of Primes

Where are the Primes?

As the prime numbers are infinite, so whatever the composite number, there is always an integer number C such that:

$$Prime = n(n \pm k) + C$$

Except for the first prime, all Primes are Odd numbers.

Thus, an "almost" general equation for all prime numbers is

$$Prime\ Number = (6m \pm 1) = n * (n + k) + C \neq n * (n + k)$$

where $n \neq 1;\ k \neq -n$ and $(m, n, k, C) \in \mathbb{Z}$.

The equation is almost general because it rules out Primes 2 and 3. A single detail that embedded all meanings of Prime numbers.

If we consider that in our sequence of prime numbers all are Odd, then the difference between two consecutive elements will be an Even number.

Thus, being our sequence composed of 3 consecutive prime numbers $P1, P2, P3$, such that $2 < P1 < P2 < P3$, then

$$P2 \geq P1 + 2\ and\ P3 \geq P2 + 2$$

Therefore,

$$P3 \geq P1 + 4$$

Next, consider a fourth Prime number P4 greater than P3:

$$P4 \geq P1 + 6$$

This means that, if there is a sequence of Odd numbers where each number is a Prime and the difference between two sequential numbers is 2, then the following property would apply:

$$P1 = P1 + 0$$
$$P2 = P1 + 2$$
$$P3 = P1 + 4$$
$$P4 = P1 + 6$$
$$P5 = P1 + 8$$
$$\dots$$

If this reasoning were correct, then we could state that "every even number is the result of the difference of two prime numbers, where one of them is fixed".

Based upon this reasoning, a table starting with the positive Integers can see where the sequences of Prime numbers appear:

| C | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 |
| 3 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 |
| 4 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 5 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 |
| 6 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 |
| 7 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |
| 8 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 |
| 9 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 10 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 |
| 11 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 |
| 12 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 |
| 13 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 |
| 14 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 15 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 |
| 16 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 |
| 17 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 |
| 18 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 |
| 19 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 20 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 |
| 21 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 |
| 22 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 |
| 23 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 |
| 24 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 25 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 |
| 26 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 |
| 27 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 |
| 28 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 |
| 29 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 30 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 |
| 31 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 |
| 32 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 |
| 33 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 |
| 34 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |
| 35 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 |
| 36 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 |
| 37 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 |
| 38 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 |
| 39 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 |
| 40 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 |
| 41 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 |

Figure 1. The sequences of Primes in blue color. They were obtained by adding the Even numbers $(0,2,4,6,8,10,12,14,\dots)$ to the Natural numbers from Zero to 41.

The only sequence larger than twin Primes is a 3-element sequence $\{3,5,7\}$. All the other Prime sequences are twin Prime numbers.

We can explain the lack of Prime sequences greater than twin Primes except for the sequences $\{|-7|,|-5|,|-3|\}\ and\ \{3,5,7\}$.

Since we are creating sequences with consecutive elements that have constant difference 2 between them, so we create two types of sequences. If the sequence starts with an Even number, we create the sequence Even numbers (2n). If the sequence starts with an Odd number, we create the sequence Odd numbers (2n-1).

If we could apply this property to Prime numbers, then either all even numbers or all odd numbers would be Prime.

The “almost” general equation $Prime\ Number = (6m \pm 1)$ says that Primes have to be Odd. However, there are Odd Composite numbers written as $n(n-k), where\ 1 \neq n \neq k$. Therefore, not all Odd numbers are Prime.

Secondly, according to the “almost” general equation of Prime numbers, $Prime\ Number = (6m \pm 1)$, if $P2 = P1 + 2$, then, $P1 = (6n-1)$ and $P2 = (6n+1)$. If $P3 = P2 + 2$, then $P3 = (6n+1) + 2 = (6n+3)$, which cannot be a Prime because it is a multiple of 3.

## 7.2 A second trial to find sequences of Primes

Sharpening the first reasoning above, while checking for sequences of Primes only separated by Even numbers, then the task now is to figure out which sequences of Even numbers may separate Prime numbers into sequences.

According to the previously stated conclusion, there is not $P2 = P1 + 2$ so that $P3 = P2 + 2$. Taking into consideration the equation $(6n \pm 1)$, if $P1 = (6n - 1)$ is a Prime, then $P2 = P1 + 2 = (6n + 1)$ may be Prime.

Now, let's consider the following possibilities for P3:

$P3 = P2 + 2 = P1 + 4 = (6n - 1) + 4 = (6n + 3)$ cannot be Prime.

$P3 = P2 + 4 = P1 + 6 = (6n - 1) + 6 = (6n + 5) = (6m - 1)$ may be Prime.

Therefore, $P2 = P1 + 2$ and $P3 = P2 + 4 = P1 + 6$, may form a sequence of Primes $P1, P2, P3$.

$$P1 = P1 + 0$$
$$P2 = P1 + 2$$
$$P3 = P1 + 6$$

This sequence overcomes the obstacles encountered in the first reasoning. Let's continue the sequence looking for P4.

The inclusion of a fourth Prime number into this sequence requires the identification of an Even number that separates the fourth Prime from the third one.

$$P1 = (6n - 1) \textit{ may be prime}$$
$$P2 = P1 + 2 = (6n + 1) \textit{ may be prime}$$
$$P3 = P2 + 4 = (6n + 5) = (6m - 1) \textit{ may be prime}$$

Therefore, the possibilities for P4:

$$P4 = P3 + 2 = (6m - 1) + 2 = (6m + 1) \textit{ may be prime}$$
$$P4 = P3 + 4 = (6m - 1) + 4 = (6m + 3) \textit{ cannot be prime}$$
$$P4 = P3 + 6 = (6m - 1) + 6 = (6t - 1) \textit{ may be prime}$$

The remaining possibilities are:

$$P4 = P3 + 2$$
$$P4 = P3 + 6$$

If we write our sequence from $P1$ and consider $P4 = P3 + 2$, we have:

$$P1 = P1 + 0$$
$$P2 = P1 + 2$$
$$P3 = P1 + 6$$
$$P4 = P1 + 8$$

If we write our sequence from $P1$ and consider $P4 = P3 + 6$, we have:

$$P1 = P1 + 0$$
$$P2 = P1 + 2$$
$$P3 = P1 + 6$$
$$P4 = P1 + 12$$

Which of the two possibilities is to be chosen now?

The possible sums of even numbers to the initial Prime number of type P1=(6n-1) are shown here where the result of P4 will never be a multiple of 2 or 3. Therefore, in terms of Primes 2 and 3, either P4=P1+8 or P4=P1+12 are appropriate.

But what would be the behavior of this sequence in terms of the Prime 5?

If P1 = (6n-1), then (6n-1) is always an Odd number with unit digit 1,3,5,7 or 9. We discard the numbers with the unit digit 5 for Primes >5.

As we write the Prime number in the forms (6n-1) or (6n+1), we must investigate all possibilities of a unit digit for each possible Prime:

| n | P1 = (6n-1) |
|---|---|
| 1 | 5 |
| 2 | 11 |
| 3 | 17 |
| 4 | 23 |
| 5 | 29 |
| 6 | 35 |
| 7 | 41 |
| 8 | 47 |
| 9 | 53 |
| 10 | 59 |
| 11 | 65 |
| 12 | 71 |
| 13 | 77 |
| 14 | 83 |
| 15 | 89 |
| 16 | 95 |
| 17 | 101 |
| 18 | 107 |
| 19 | 113 |
| 20 | 119 |
| 21 | 125 |

| n | P1 = (6n+1) |
|---|---|
| 1 | 7 |
| 2 | 13 |
| 3 | 19 |
| 4 | 25 |
| 5 | 31 |
| 6 | 37 |
| 7 | 43 |
| 8 | 49 |
| 9 | 55 |
| 10 | 61 |
| 11 | 67 |
| 12 | 73 |
| 13 | 79 |
| 14 | 85 |
| 15 | 91 |
| 16 | 97 |
| 17 | 103 |
| 18 | 109 |
| 19 | 115 |
| 20 | 121 |
| 21 | 127 |

Table 1. The Prime numbers forms (6n-1) or (6n+1)

All Prime numbers, except Primes 2 and 5, have the unit digits {1, 3, 7, 9}.

Now let's add each of the even number sequences we find in each digit unit of P1=(6n-1).

| Possibles sequences with (P4 = P1+8) mod 10 | | | | |
|---|---|---|---|---|
| (P1 = P1+0) mod 10 | 1 | 3 | 7 | 9 |
| (P2 = P1+2) mod 10 | 3 | 5 | 9 | 1 |
| (P3 = P1+6) mod 10 | 7 | 9 | 3 | 5 |
| (P4 = P1+8) mod 10 | 9 | 1 | 5 | 7 |
| | good | bad | bad | bad |

Table 1. The sum of the Even sequence (0,2,6,8) in each of the four unit digits of P1.

| Possibles sequences with (P4 = P1+12) mod 10 | | | | |
|---|---|---|---|---|
| (P1 = P1+0) mod 10 | 1 | 3 | 7 | 9 |
| (P2 = P1+2) mod 10 | 3 | 5 | 9 | 1 |
| (P3 = P1+6) mod 10 | 7 | 9 | 3 | 5 |
| (P4 = P1+12) mod 10 | 3 | 5 | 9 | 1 |
| | good | bad | good | bad |

Table 1. The sum of the Even sequence (0,2,6,12) in each of the four unit digits of P1.

If we use P4=P1+8, then we can only get prime number sequences from P1 with unit digits {1}.

However, if we use P4=P1+12, then we can get prime number sequences from P1 with unit digits {1 and 7}.

This means we should choose P4=P1+12 to cover more Prime number sequences.

So, our choice is:

$$P1 = P1 + 0$$

$$P2 = P1 + 2$$
$$P3 = P1 + 6$$
$$P4 = P1 + 12$$

Now that we have determined four possible Prime elements of our sequence, what should the next Prime elements be like?

The most intuitive answer is to verify the mechanism that exists between the differences of the sums to extrapolate ahead.

We are adding (0,2,6,12). The difference between them is (2,4,6). We are increasing by 2 each of the differences:

| Possible Prime Sequences at P1 mod 10 = (1 or 7) | | | |
|---|---|---|---|
| | | dif | difdif |
| P2=P1+ | 2 | 2 | 2 |
| P3=P1+ | 6 | 4 | 2 |
| P4=P1+ | 12 | 6 | 2 |
| P5=P1+ | 20 | 8 | 2 |
| P6=P1+ | 30 | 10 | 2 |
| P7=P1+ | 42 | 12 | 2 |
| P8=P1+ | 56 | 14 | 2 |
| P9=P1+ | 72 | 16 | 2 |

Table 1. To get the greatest success of Prime number sequences we need to add to an initial Prime number of form $P1 = (6n \pm 1)$ the even number sequence $(0,2,6,12,20,30,42,56,72,\dots)$. With this procedure, we avoid finding multiples of 2, 3, and 5.

Let's test this theory in practice.

Let's a row of all Natural numbers including Zero and One.

- For each Natural number from this first row, we create a second row below where each element will be the elements from the first row by adding 2.
- For each Natural number from this second row, we create a third row below where each element will be the elements from the second row by adding 4.
- For each Natural number from this third row, we create a fourth row below where each element will be the elements from the third row by adding 6.
- For each Natural number from this fourth row, we create a fifth row below where each element will be the elements from the fourth row by adding 8. And so on…

So,

- The first row will be the Natural numbers.
- The second row will be the first row of Natural numbers added by 2.
- The third row will be the first row of Natural numbers added by 6.
- The fourth row will be the first row of Natural numbers added by 12.
- The fifth row will be the first row of Natural numbers added by 20. And so on…

See what a beautiful and intriguing table we get:

| dif d | dif | C | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 |
| | 2 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 |
| 2 | 4 | 3 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 |
| 2 | 6 | 4 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 |
| 2 | 8 | 5 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 |
| 2 | 10 | 6 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 |
| 2 | 12 | 7 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 |
| 2 | 14 | 8 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 |
| 2 | 16 | 9 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 |
| 2 | 18 | 10 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 | 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 |
| 2 | 20 | 11 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 | 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 | 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 | 151 |
| 2 | 22 | 12 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 | 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 | 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 | 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 | 171 | 172 | 173 |
| 2 | 24 | 13 | 156 | 157 | 158 | 159 | 160 | 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 | 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 | 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 | 191 | 192 | 193 | 194 | 195 | 196 | 197 |
| 2 | 26 | 14 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 | 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 | 201 | 202 | 203 | 204 | 205 | 206 | 207 | 208 | 209 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 | 223 |
| 2 | 28 | 15 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 | 223 | 224 | 225 | 226 | 227 | 228 | 229 | 230 | 231 | 232 | 233 | 234 | 235 | 236 | 237 | 238 | 239 | 240 | 241 | 242 | 243 | 244 | 245 | 246 | 247 | 248 | 249 | 250 | 251 |
| 2 | 30 | 16 | 240 | 241 | 242 | 243 | 244 | 245 | 246 | 247 | 248 | 249 | 250 | 251 | 252 | 253 | 254 | 255 | 256 | 257 | 258 | 259 | 260 | 261 | 262 | 263 | 264 | 265 | 266 | 267 | 268 | 269 | 270 | 271 | 272 | 273 | 274 | 275 | 276 | 277 | 278 | 279 | 280 | 281 |
| 2 | 32 | 17 | 272 | 273 | 274 | 275 | 276 | 277 | 278 | 279 | 280 | 281 | 282 | 283 | 284 | 285 | 286 | 287 | 288 | 289 | 290 | 291 | 292 | 293 | 294 | 295 | 296 | 297 | 298 | 299 | 300 | 301 | 302 | 303 | 304 | 305 | 306 | 307 | 308 | 309 | 310 | 311 | 312 | 313 |
| 2 | 34 | 18 | 306 | 307 | 308 | 309 | 310 | 311 | 312 | 313 | 314 | 315 | 316 | 317 | 318 | 319 | 320 | 321 | 322 | 323 | 324 | 325 | 326 | 327 | 328 | 329 | 330 | 331 | 332 | 333 | 334 | 335 | 336 | 337 | 338 | 339 | 340 | 341 | 342 | 343 | 344 | 345 | 346 | 347 |
| 2 | 36 | 19 | 342 | 343 | 344 | 345 | 346 | 347 | 348 | 349 | 350 | 351 | 352 | 353 | 354 | 355 | 356 | 357 | 358 | 359 | 360 | 361 | 362 | 363 | 364 | 365 | 366 | 367 | 368 | 369 | 370 | 371 | 372 | 373 | 374 | 375 | 376 | 377 | 378 | 379 | 380 | 381 | 382 | 383 |
| 2 | 38 | 20 | 380 | 381 | 382 | 383 | 384 | 385 | 386 | 387 | 388 | 389 | 390 | 391 | 392 | 393 | 394 | 395 | 396 | 397 | 398 | 399 | 400 | 401 | 402 | 403 | 404 | 405 | 406 | 407 | 408 | 409 | 410 | 411 | 412 | 413 | 414 | 415 | 416 | 417 | 418 | 419 | 420 | 421 |
| 2 | 40 | 21 | 420 | 421 | 422 | 423 | 424 | 425 | 426 | 427 | 428 | 429 | 430 | 431 | 432 | 433 | 434 | 435 | 436 | 437 | 438 | 439 | 440 | 441 | 442 | 443 | 444 | 445 | 446 | 447 | 448 | 449 | 450 | 451 | 452 | 453 | 454 | 455 | 456 | 457 | 458 | 459 | 460 | 461 |
| 2 | 42 | 22 | 462 | 463 | 464 | 465 | 466 | 467 | 468 | 469 | 470 | 471 | 472 | 473 | 474 | 475 | 476 | 477 | 478 | 479 | 480 | 481 | 482 | 483 | 484 | 485 | 486 | 487 | 488 | 489 | 490 | 491 | 492 | 493 | 494 | 495 | 496 | 497 | 498 | 499 | 500 | 501 | 502 | 503 |
| 2 | 44 | 23 | 506 | 507 | 508 | 509 | 510 | 511 | 512 | 513 | 514 | 515 | 516 | 517 | 518 | 519 | 520 | 521 | 522 | 523 | 524 | 525 | 526 | 527 | 528 | 529 | 530 | 531 | 532 | 533 | 534 | 535 | 536 | 537 | 538 | 539 | 540 | 541 | 542 | 543 | 544 | 545 | 546 | 547 |
| 2 | 46 | 24 | 552 | 553 | 554 | 555 | 556 | 557 | 558 | 559 | 560 | 561 | 562 | 563 | 564 | 565 | 566 | 567 | 568 | 569 | 570 | 571 | 572 | 573 | 574 | 575 | 576 | 577 | 578 | 579 | 580 | 581 | 582 | 583 | 584 | 585 | 586 | 587 | 588 | 589 | 590 | 591 | 592 | 593 |
| 2 | 48 | 25 | 600 | 601 | 602 | 603 | 604 | 605 | 606 | 607 | 608 | 609 | 610 | 611 | 612 | 613 | 614 | 615 | 616 | 617 | 618 | 619 | 620 | 621 | 622 | 623 | 624 | 625 | 626 | 627 | 628 | 629 | 630 | 631 | 632 | 633 | 634 | 635 | 636 | 637 | 638 | 639 | 640 | 641 |
| 2 | 50 | 26 | 650 | 651 | 652 | 653 | 654 | 655 | 656 | 657 | 658 | 659 | 660 | 661 | 662 | 663 | 664 | 665 | 666 | 667 | 668 | 669 | 670 | 671 | 672 | 673 | 674 | 675 | 676 | 677 | 678 | 679 | 680 | 681 | 682 | 683 | 684 | 685 | 686 | 687 | 688 | 689 | 690 | 691 |
| 2 | 52 | 27 | 702 | 703 | 704 | 705 | 706 | 707 | 708 | 709 | 710 | 711 | 712 | 713 | 714 | 715 | 716 | 717 | 718 | 719 | 720 | 721 | 722 | 723 | 724 | 725 | 726 | 727 | 728 | 729 | 730 | 731 | 732 | 733 | 734 | 735 | 736 | 737 | 738 | 739 | 740 | 741 | 742 | 743 |
| 2 | 54 | 28 | 756 | 757 | 758 | 759 | 760 | 761 | 762 | 763 | 764 | 765 | 766 | 767 | 768 | 769 | 770 | 771 | 772 | 773 | 774 | 775 | 776 | 777 | 778 | 779 | 780 | 781 | 782 | 783 | 784 | 785 | 786 | 787 | 788 | 789 | 790 | 791 | 792 | 793 | 794 | 795 | 796 | 797 |
| 2 | 56 | 29 | 812 | 813 | 814 | 815 | 816 | 817 | 818 | 819 | 820 | 821 | 822 | 823 | 824 | 825 | 826 | 827 | 828 | 829 | 830 | 831 | 832 | 833 | 834 | 835 | 836 | 837 | 838 | 839 | 840 | 841 | 842 | 843 | 844 | 845 | 846 | 847 | 848 | 849 | 850 | 851 | 852 | 853 |
| 2 | 58 | 30 | 870 | 871 | 872 | 873 | 874 | 875 | 876 | 877 | 878 | 879 | 880 | 881 | 882 | 883 | 884 | 885 | 886 | 887 | 888 | 889 | 890 | 891 | 892 | 893 | 894 | 895 | 896 | 897 | 898 | 899 | 900 | 901 | 902 | 903 | 904 | 905 | 906 | 907 | 908 | 909 | 910 | 911 |
| 2 | 60 | 31 | 930 | 931 | 932 | 933 | 934 | 935 | 936 | 937 | 938 | 939 | 940 | 941 | 942 | 943 | 944 | 945 | 946 | 947 | 948 | 949 | 950 | 951 | 952 | 953 | 954 | 955 | 956 | 957 | 958 | 959 | 960 | 961 | 962 | 963 | 964 | 965 | 966 | 967 | 968 | 969 | 970 | 971 |
| 2 | 62 | 32 | 992 | 993 | 994 | 995 | 996 | 997 | 998 | 999 | 1000 | 1001 | 1002 | 1003 | 1004 | 1005 | 1006 | 1007 | 1008 | 1009 | 1010 | 1011 | 1012 | 1013 | 1014 | 1015 | 1016 | 1017 | 1018 | 1019 | 1020 | 1021 | 1022 | 1023 | 1024 | 1025 | 1026 | 1027 | 1028 | 1029 | 1030 | 1031 | 1032 | 1033 |
| 2 | 64 | 33 | 1056 | 1057 | 1058 | 1059 | 1060 | 1061 | 1062 | 1063 | 1064 | 1065 | 1066 | 1067 | 1068 | 1069 | 1070 | 1071 | 1072 | 1073 | 1074 | 1075 | 1076 | 1077 | 1078 | 1079 | 1080 | 1081 | 1082 | 1083 | 1084 | 1085 | 1086 | 1087 | 1088 | 1089 | 1090 | 1091 | 1092 | 1093 | 1094 | 1095 | 1096 | 1097 |
| 2 | 66 | 34 | 1122 | 1123 | 1124 | 1125 | 1126 | 1127 | 1128 | 1129 | 1130 | 1131 | 1132 | 1133 | 1134 | 1135 | 1136 | 1137 | 1138 | 1139 | 1140 | 1141 | 1142 | 1143 | 1144 | 1145 | 1146 | 1147 | 1148 | 1149 | 1150 | 1151 | 1152 | 1153 | 1154 | 1155 | 1156 | 1157 | 1158 | 1159 | 1160 | 1161 | 1162 | 1163 |
| 2 | 68 | 35 | 1190 | 1191 | 1192 | 1193 | 1194 | 1195 | 1196 | 1197 | 1198 | 1199 | 1200 | 1201 | 1202 | 1203 | 1204 | 1205 | 1206 | 1207 | 1208 | 1209 | 1210 | 1211 | 1212 | 1213 | 1214 | 1215 | 1216 | 1217 | 1218 | 1219 | 1220 | 1221 | 1222 | 1223 | 1224 | 1225 | 1226 | 1227 | 1228 | 1229 | 1230 | 1231 |
| 2 | 70 | 36 | 1260 | 1261 | 1262 | 1263 | 1264 | 1265 | 1266 | 1267 | 1268 | 1269 | 1270 | 1271 | 1272 | 1273 | 1274 | 1275 | 1276 | 1277 | 1278 | 1279 | 1280 | 1281 | 1282 | 1283 | 1284 | 1285 | 1286 | 1287 | 1288 | 1289 | 1290 | 1291 | 1292 | 1293 | 1294 | 1295 | 1296 | 1297 | 1298 | 1299 | 1300 | 1301 |
| 2 | 72 | 37 | 1332 | 1333 | 1334 | 1335 | 1336 | 1337 | 1338 | 1339 | 1340 | 1341 | 1342 | 1343 | 1344 | 1345 | 1346 | 1347 | 1348 | 1349 | 1350 | 1351 | 1352 | 1353 | 1354 | 1355 | 1356 | 1357 | 1358 | 1359 | 1360 | 1361 | 1362 | 1363 | 1364 | 1365 | 1366 | 1367 | 1368 | 1369 | 1370 | 1371 | 1372 | 1373 |
| 2 | 74 | 38 | 1406 | 1407 | 1408 | 1409 | 1410 | 1411 | 1412 | 1413 | 1414 | 1415 | 1416 | 1417 | 1418 | 1419 | 1420 | 1421 | 1422 | 1423 | 1424 | 1425 | 1426 | 1427 | 1428 | 1429 | 1430 | 1431 | 1432 | 1433 | 1434 | 1435 | 1436 | 1437 | 1438 | 1439 | 1440 | 1441 | 1442 | 1443 | 1444 | 1445 | 1446 | 1447 |
| 2 | 76 | 39 | 1482 | 1483 | 1484 | 1485 | 1486 | 1487 | 1488 | 1489 | 1490 | 1491 | 1492 | 1493 | 1494 | 1495 | 1496 | 1497 | 1498 | 1499 | 1500 | 1501 | 1502 | 1503 | 1504 | 1505 | 1506 | 1507 | 1508 | 1509 | 1510 | 1511 | 1512 | 1513 | 1514 | 1515 | 1516 | 1517 | 1518 | 1519 | 1520 | 1521 | 1522 | 1523 |
| 2 | 78 | 40 | 1560 | 1561 | 1562 | 1563 | 1564 | 1565 | 1566 | 1567 | 1568 | 1569 | 1570 | 1571 | 1572 | 1573 | 1574 | 1575 | 1576 | 1577 | 1578 | 1579 | 1580 | 1581 | 1582 | 1583 | 1584 | 1585 | 1586 | 1587 | 1588 | 1589 | 1590 | 1591 | 1592 | 1593 | 1594 | 1595 | 1596 | 1597 | 1598 | 1599 | 1600 | 1601 |
| 2 | 80 | 41 | 1640 | 1641 | 1642 | 1643 | 1644 | 1645 | 1646 | 1647 | 1648 | 1649 | 1650 | 1651 | 1652 | 1653 | 1654 | 1655 | 1656 | 1657 | 1658 | 1659 | 1660 | 1661 | 1662 | 1663 | 1664 | 1665 | 1666 | 1667 | 1668 | 1669 | 1670 | 1671 | 1672 | 1673 | 1674 | 1675 | 1676 | 1677 | 1678 | 1679 | 1680 | 1681 |
| 2 | 82 | 42 | 1722 | 1723 | 1724 | 1725 | 1726 | 1727 | 1728 | 1729 | 1730 | 1731 | 1732 | 1733 | 1734 | 1735 | 1736 | 1737 | 1738 | 1739 | 1740 | 1741 | 1742 | 1743 | 1744 | 1745 | 1746 | 1747 | 1748 | 1749 | 1750 | 1751 | 1752 | 1753 | 1754 | 1755 | 1756 | 1757 | 1758 | 1759 | 1760 | 1761 | 1762 | 1763 |

Figure 1. The sequences of Primes in blue color. They were obtained by adding the Even numbers $(0,2,6,12,20,30,42,56,72,\dots)$ to the Natural numbers from Zero to 41.

When we look at this table, several questions arose.

- What does it mean? What are the Prime number sequences talking about here? What do they want to show?
- How far do sequences appear? Why do they appear in this way?
- Where are the sequences of different initial Prime numbers? Beautiful Prime sequences are seen for columns 5, 11, 17, and 41, but where are the sequences of 7, 13, 19, and 43? And those of other Primes?
- Why do Prime sequences stop?
- Are there infinite sequences of Primes? If we continue expanding the table, do we find a sequence with infinite many Primes? Can we find a sequence with as many Primes as we wish?
- What happens if the table is extended to the other 3 quadrants using the negative columns and negative rows? Is it possible?
- Are there more sequences of "hidden" Primes sequences other than the obvious vertical sequences?
- Why do some Primes appear in more than one sequence?
- Why are they aligned? What are the rules for this alignment?
- Is it possible to produce new tables and get new sequences?
- ...

Here we start all our studies.

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## References

[1] *The On-Line Encyclopedia of Integer Sequences*, available online at http://oeis.org.

[2]