Integrated Shipment and Production Planning: a Mixed-Integer Linear Programming Approach with Multiple Suppliers and Due Dates

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November 28, 2023

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***Abstract*—Planning of production orders in an assembly-based industry highly depends on the available components. Typically, the production plan is followed by a material requirement planning to ensure the availability of the required component and order the needed quantities from suppliers. This is why routing these elements and their availability greatly impacts the production plan and vice versa. The coordination of both plans simultaneously can improve the quality of the planning decisions and reduce operational costs. This research studies the effect of integrating routing decisions with the production plan considering environmental impact. We developed a mixed integer linear programming model; the objective function of the problems includes the operational inbound routing costs, the holding cost of components in the manufacturing site's inventory, the lateness cost of the production order, and minimizing greenhouse gas emissions. Our study is exemplified in a numerical example where we present three different scenarios, highlighting how prioritizing the manufacturing site's different objectives can directly impact the production plan of the manufacturing orders.**

***Index Terms*—Integrated Planning, Supply-Production Planning, Optimisation, Carbon Emission**

I. Introduction

Production scheduling and vehicle routing have been extensively examined separately. Traditional methods of solving these problems independently have demonstrated limitations in optimizing overall performance. Hence, a synchronized approach that addresses these interrelated problems concurrently becomes interesting to many industries. In the automotive industry, embracing an integrated approach is vital in today's fiercely competitive business landscape as it enables cost savings and ensures timely deliveries are consistently achieved [4]. In addition, the importance of reducing emissions is increasing, intending to eliminate them in the coming years largely, and most companies have objectives to reduce their greenhouse gas emissions with objectives and milestones to meet by as near as 2025.

To meet these goals, we consider a comprehensive inbound logistics and production planning problem taking into account four different modes of transport Road Transport such as standard trucks (tr) and express trucks (ces); maritime transport, such as boats (bt); and aerial transport, such as cargo planes (ap). We have developed a mathematical model to address the problem at hand. Our formulated model takes the form of a mixed-integer linear program (MILP), which enables optimization by considering the selection of the most suitable transport mode from the suppliers, be it one of the four modes earlier presented. Simultaneously, the model determines the optimal component supply quantities and the optimal production plan while respecting the presented industrial constraints, such as the manufacturing site's maximum production capacity.

Furthermore, our model considers the manufacturing site's holding capacity for the components, planned delivery schedules from the suppliers, greenhouse gas emissions, and due dates of the manufacturing orders. Through utilising our MILP-based model, decision-makers can make informed choices regarding the selection of the inbound transport modes, determine optimal component quantities, and design efficient transportation patterns while considering storage utilization and handling capacities. By integrating these considerations into a comprehensive optimization framework, our model provides decision support for enhancing supply chain operations' overall performance and efficiency.

The paper is organized as follows. First, an overview of relevant literature (section II). Second, we present the problem statement (section III); the mathematical model is then presented (section IV) with a numerical example (section

V) where we present three different scenarios of supply and production plan and discussion of the results and summary of the main findings in (section VI).

II. Litterature Review

In supply chain management, a complex system encompasses numerous functions, activities, and organizational entities. This inherent complexity poses a substantial challenge to achieving optimal overall performance. Addressing these concerns necessitates effective coordination within the supply chain [11], [4]. Extensive research has revealed that integrating production, inventory, and distribution or routing decisions within supply chains presents significant opportunities for cost savings and enhanced operational efficiency for organizations [4]. Significant research has focused on integrating production and inbound logistics planning in recent decades. However, most articles emphasize this integration's strategic or tactical aspects. Adulyasaketal. (2015), D. Hrabec et al. (2022), and L. Berghman et al. (2023) [11], [4], [6] have provided comprehensive summaries and reviews of existing formulations and solution techniques for these production routing problems.

Integrating production scheduling and vehicle routing can be traced back to the 1950s when A.J. Clark (1958) [5] defined the basic framework for an integrated supply chain. This early work extended the inventory problem and economic order quantity into a single-item multi-echelon inventory problem.

Our research has uncovered notable contributions in this area. B. R. Sarker and A. Diponegoro (2009) [9] focused on developing an optimal production and procurement policy for a multi-supplier, manufacturer, and buyer supply-chain system. Garcia-Sabater et al. (2013) [1] introduced the concept of the generic materials and operations planning (GMOP) problem, which is built upon the notion of 'strokes'. In contrast to conventional methods that prioritize materials or resources as decision variables, the GMOP model takes a different approach by emphasizing the operations (strokes) that each resource can perform.

H.M. Afsar and F. Hnaien (2020) [7] presented a study that focuses on the dynamic version of the assembly routing problem, addressing both production decisions and material replenishment in an integrated manner. Three linear programming models are proposed, including a non-vehicle index model and a logic-based benders decomposition approach. F. Hein & C. Aldemer (2016) [3] emphasized the significance of coordination, showcasing that as the problem size increases, the value of integrating inbound logistics and production planning grows. And companies that adopt the just-in-time (JIT) approach—a lean manufacturing method where components are delivered precisely when needed, avoiding excessive inventory holding can expect even greater gains from integrating these functions.

M. Salehi Sarbijan and J. Behnamian (2021) [8] explored the integration of production routing decisions with outsourcing and environmental considerations, using a mixed-integer linear programming model and a particle swarm optimization algorithm to minimize costs and greenhouse gas emissions. C. Wang et al. (2023) [10] developed an uncertain programming model and investigated different replenishment policies for the integrated production routing problem under uncertain demands, highlighting the impact of confidence levels and variances of uncertain variables on overall costs.

M. Quetschlich et al. (2021) [2] provided a generic model capable of combining any production, inventory, and transportation element. Their work highlights the need for mathematical models that address routing products, sub-products, and raw materials with a nested multi-level bill of materials (BOMs) and transportation equipment across multi-echelon supply networks. In response to this gap, their study presents a generic optimization model that serves as a reference for routing these complex products through intricate supply chains.

Many works studied the integration of supply and production planning, and our work contributes to the work in the literature with the following: (1) We present a supply-production optimization problem considering greenhouse gas emissions. (2) We compare scenarios to highlight how the company policy can impact the manufacturing site's tactical-level planning.

III. Problem Definition

Figure 1 represents our study's integrated supply, production, and inventory system. The system comprises a single manufacturing site, one inventory (for the components), several suppliers, and a transportation fleet responsible for transferring materials between the supplier and the manufacturing site; the fleet comprises different transportation modes; trucks, express trucks, boats and cargo-planes. For example, supplier 3 provides a component that can only be delivered by maritime transport.

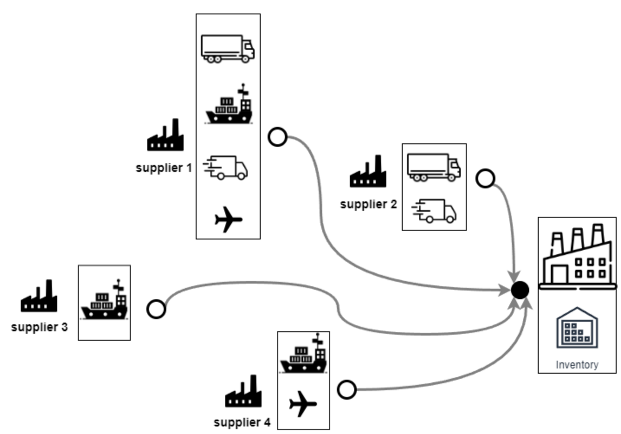

Fig. 1. Illustrative example of the supply chain network studied.

In this system, the manufacturing site tries to find the optimal production and inbound supply plans while respecting the site's constraints and the demand due dates. The production quantities per period are subject to a finite capacity constraint, which limits the amount that can be produced per period. The process starts with determining the components required to produce manufacturing orders. The manufacturing site must collect components from various suppliers to replenish the input material inventories. Each supplier offers multiple supply options that differ in quantity, lead time, cost, and average greenhouse gas emissions. The quantity of components supplied depends on the transportation mode. For example, an express truck carries less quantities than a standard truck. The delivery lead time depends on the supplier's distance from the manufacturing site and transportation mode. The supply cost depends on the quantity and the transportation mode used. Finally, the greenhouse gas emissions are calculated based on the supplier's distance from the manufacturing site and the transportation mode. For example, a supplier close to the manufacturing site can offer supply with a standard truck or a cargo plane; the manufacturing site must choose which transportation means to replenish their inventories. Their decision varies depending on the delays they could allow themselves without impacting the due dates on the manufacturing orders. The primary objective of the planning problem described is to minimize the total cost associated with the system. These costs include several components: holding costs for components in the inventories, transportation costs, lateness costs, and greenhouse gas emissions costs. The holding cost for components reflects the costs incurred in storing and managing the components required to produce the manufacturing orders. It considers storage capacity, inventory control, and the potential risks of carrying excess or obsolete components. The transportation cost accounts for the expenses related to the supply of components between the suppliers and the manufacturing site. The lateness cost refers to the penalty of not respecting the manufacturing order's due dates, and these penalties are exemplified in contractual penalties and the degradation of customer satisfaction. Finally, the greenhouse gas emissions costs are calculated only from the supply side. The costs are the average emissions based on the component size and quantity, transportation mode, and distance travelled. The overall objective of the planning problem is to find an optimal solution that minimizes the cumulative cost of these components.

IV. Mathematical Formulation

We have the following sets and indices:

$o \in \mathcal{O}$:	set of orders.
$d \in \mathcal{D}$:	set of demands.
$p \in \mathcal{P}$:	set of products.
$t \in \mathcal{T}$:	set of planning periods.
$k \in \mathcal{K}$:	set of components.
$s \in \mathcal{S}$:	set of suppliers.
$w \in \mathcal{W}$:	set of transport mode.

The parameters introduced in this model can be categorized into input data such as demand, delays and constraint-related parameters such as maximum production capacity.

$Dem_{o,d}$:	quantity of demand of order o and demand d.
$DD_{o,d}$:	due date t of demand d of order o.
$Pr_{o,d}$:	priority of demand d of order o.
$D_{i,w}$:	quantity of demand of product type i and priority w.
$Pr_{o,d}$:	product type i of demand d of order o.
G^i_k:	BOM, quantity of component k needed to build product i.
I^k:	initial Inventory of components at $t = t1$.
$\mathcal{Q}^{w,s}_k$:	supply quantity of a shipment of component k from supplier s with transport mode w.
h_k:	holding cost of components k per period.
pc_1_t:	production capacity of the manufacturing site in period t.
$pc\_2^i_t$:	production capacity of the manufacturing site in period t for product type i.
ds^k_t:	quantity of component k shipment to be delivered in period t.
Max_inv_k:	maximum capacity of inventory for part k.
Min_inv_k:	minimum inventory to keep at the end of period t.
$sc^{w,s}_k$:	cost of emergency shipment of part k using transport mode w.
$lt^{w,s}_k$:	delivery lead time of component k from supplier s with transport mode w.
$co^{w,s}_k$:	greenhouse gas emissions of part k supply with transport mode w from supplier s.
$\beta_{o,d}(t, \mathcal{DD}_{o,d})$:	tardiness cost of demand d of order o for producing at t later than $\mathcal{DD}_{o,d}$.
$C(t)^{o,d}$:	lateness cost of producing the demand d of order o in the period t.

The decision variables in this model can be classified into three categories: manufacturing-related variables determining the production schedule, logistics-related variables governing inventory management, and supply-related variables considering delivery constraints. We have the following decision variables:

$m_{d,o,t}$:	quantity of manufacturing orders to produce for each demand d of order o at period t.
$ls_{d,o,t}$:	lost sales in the studied horizon $\mathcal{T}$.
$cp_{k,t}$:	quantity of components consumed over a period t.
$Inv_{k,t}$:	the inventory level of each component k at the end of period t.
$sp^{w,s}_{k,t}$:	orders of replenishment over period t.
$ap_{k,t}$:	a binary flag equals 1 if a purchase of component k was done by a cargo plane at period t.

The objective function minimizes the sum of holding costs of components and transportation costs of supply the lateness and the greenhouse gas emissions over the entire planning horizon.

$$Min\ c1 + c2 + c3 + c4$$

Cost 1- Lateness cost: The lateness cost function $\beta_{o,d}(t, \mathcal{DD}_{o,d})$ is monotonically increasing if the number of late periods increases (i.e. $t - \mathcal{DD}_{o,d}$), the tardiness function is linear and represented in equation 2, where μ, η are coefficients with predefined values.

$$\mathcal{C}(t)^{o,d} = \begin{cases} \beta_{o,d}(t+1, \mathcal{DD}_{o,d}) - \quad \beta_{o,d}(t, \mathcal{DD}_{o,d}) \\ \qquad\qquad\qquad\qquad\qquad \text{if } t > \mathcal{DD}_{o,d}, \\ 0 \qquad\qquad\qquad\qquad\quad \text{Otherwise} \end{cases} \tag{1}$$

$$\beta_{o,d}(t, \mathcal{DD}_{o,d}) = \mu * (t - \mathcal{DD}_{o,d}) + \eta * (\mathcal{DD}_{o,d} - t0)^2 \tag{2}$$

Finally, to calculate the total lateness cost, we use the equation presented in 3.

$$\sum_{d\in\mathcal{D}}\sum_{o\in\mathcal{O}}\sum_{t\in\mathcal{T}}(\mathcal{C}(t)^{o,d} * m_{d,o,t}) \tag{3}$$

Cost 2- Holding of components: components are replenished and stored in the manufacturing site inventory, and each component k has a defined holding cost h_k. Equation 4 presents how this cost is calculated.

$$\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}(h_k * \mathrm{Inv}_{k,t}) \tag{4}$$

Cost 3- Supply Cost: The supply (replenishment) cost of component k varies depending on the quantity and the transportation mode. Equation 5 presents how this cost is calculated.

$$\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}}\sum_{s\in\mathcal{S}}(\mathrm{sc}_k^{w,s} * \mathrm{sp}_{k,t}^{w,s}) \tag{5}$$

Cost 4- Greenhouse gas emissions: The greenhouse gas emissions cost is calculated based on the supplier's geographical distance from the manufacturing site and the transportation mode. The emission is proportional to the quantity being shipped for the cargo plane. Equation 6 presents how this cost is calculated.

$$\begin{aligned}&\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}}\sum_{s\in\mathcal{S}}(\mathrm{co}_k^{w,s} * \mathrm{sp}_{k,t}^{w,s})\\ +&\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}}\sum_{s\in\mathcal{S}}(\psi{*}\mathrm{co}_k^{w,s}{*}\mathrm{sp}_{k,t}^{w,s}{*}\mathrm{Q}_k^{w,s}\ if\ w == plane)\end{aligned} \tag{6}$$

Factors like production capacity, component inventory, and storage limitations in the garage for completed orders constrain the manufacturing site. The model incorporates continuity constraints to balance the garage's component consumption, order production, and storage of completed orders. The objective function is subject to the following constraints:

Production capacity constraint: The maximum production capacity in the manufacturing site.

$$\sum_{d\in\mathcal{D}}\sum_{o\in\mathcal{O}} m_{d,o,t} \le pc\_1_t, \forall t \in \mathcal{T},$$
$$\sum_{d\in\mathcal{D}}\sum_{o\in\mathcal{O}} m_{d,o,t} \le pc\_2_{i,t}, \forall t \in \mathcal{T}, i \in \mathcal{P},\ \text{if}\ (i = Pr_{o,d})$$

Demand constraint: orders produced should not exceed the demand.

$$\sum_{t\in\mathcal{T}} m_{d,o,t} \le \mathcal{D}em_{o,d}, \forall d \in \mathcal{D}, o \in \mathcal{O}$$

Balance Constraints: continuity constraints that ensure the correct balance between inventory, consumption and supply of components.

–Components supply initial inventory balance:

$$\mathrm{Inv}_{k,t1} = ds_0^k * \mathcal{Q}_k^{w,s} + \mathrm{I}^k - \mathrm{cp}_{k,t1}$$
$$(\mathrm{lt}_k^{w,s} == 0 : + \mathrm{sp}_{k,t1}^{w,s} * \mathcal{Q}_k^{w,s}, 0)\ \forall k \in K$$

–Components supply balance:

$$\begin{aligned}\mathrm{Inv}_{k,t} = \mathrm{Inv}_{k,t-1} - \mathrm{cp}_{k,t} + (\mathrm{sp}_{k,t-(lt_k^{w,s})}^{w,s} * \mathcal{Q}_k^{w,s})\\ +(ds_t^k * \mathcal{Q}_k^{w,s})\ \forall k \in \mathcal{K}, t > t1\end{aligned}$$

Inventory capacity constraint:

$$Inv_{k,t} \le \mathrm{Max\_inv}_k\ \forall k \in \mathcal{K}, t \in \mathcal{T}$$

Inventory target constraint: To keep a minimum number of components in the inventory.

$$Inv_{k,t} \ge \mathrm{Min\_inv}_k,\ \forall k \in \mathcal{K}, t \in \mathcal{T}$$

Lost Sales constraint: Balance orders between produced or a lost sale, the sum of both should be equal to the total demand.

$$\mathrm{ls}_{d,o,t} + \sum_{t'\in[t0;t]}(\mathrm{m}_{d,o,t'}) = \mathcal{D}_{i,\mathcal{P}r_{o,d}}\ \forall o \in \mathcal{O}, d \in \mathcal{D}, t \in \mathcal{T}$$

Consume constraint: Updating components consumed at each period.

$$\mathrm{cp}_{k,t} = \sum_{d\in\mathcal{D}}\sum_{o\in\mathcal{O}} m_{d,o,t} * \mathcal{G}_k^i\ \forall t \in \mathcal{T}, k \in \mathcal{K},\ \text{if}\ (i = Pr_{o,d})$$

Emergency Shipment Flag constraint: Updating the emergency shipment flag. M is a sufficiently large value.

$$\mathrm{sp}_{k,t}^{w,s} >= ap_{k,t},\ \mathrm{sp}_{k,t}^{w,s} <= M * ap_{k,t}$$
$$\forall k \in \mathcal{K}, t \in \mathcal{T}, \forall s \in \mathcal{S}, w \in \mathcal{W},\ if(w == plane)$$

The model aims to minimize the holding cost of finished products, purchased components, and lateness-associated costs. However, not all production plans (scenarios) are equal. To evaluate and compare different production plans, we introduce the following indicators.

Service Level: As a service level indicator, we calculate the percentage of stockouts compared to total demand. This indicator translates the percentage of demand not satisfied. By designating the quantity out of stock on the reference at the end of the period, the calculation of this indicator is illustrated by the following equations:

We first study the global service level, which is the ratio of demand fulfilled over customer orders.

$$SL1 = \frac{\sum_{d\in\mathcal{D}}\sum_{o\in\mathcal{O}}\sum_{t\in\mathcal{T}} m_{d,o,t}}{\sum_{d\in\mathcal{D}}\sum_{o\in\mathcal{O}} \mathcal{D}em_{o,d}}$$

We also study the service level of each order. This allows the manufacturing site to evaluate if the service level impact affects a specific group of orders.

$$SL2(o) = \frac{\sum_{d\in\mathcal{D}}\sum_{t\in\mathcal{T}} m_{d,o,t}}{\sum_{d\in\mathcal{D}} \mathcal{D}em_{o,d}} v,\ \forall o \in \mathcal{O}$$

We then calculate the average service level per customer order. In the following section, we present a numerical example where we compare three different scenarios, highlighting how prioritizing the manufacturing site's different objectives can directly impact the production plan of the manufacturing orders.

V. Numerical Example

We consider a manufacturing site that produces two product types ('A', 'B'); the site has a manufacturing capacity of ten production orders (vehicles) per period. Each product type requires four components (parts); each component comes from a different supplier except for *'PartB'* and *'PartF'*, which come from the same supplier. A component can be installed in more than one product type. There are six components (PartA → PartF); product **A** requires *partA*, *partC*, *partD*, and *partE*, product **B** requires *partB*, *partC*, *partE*, and *partF*. We can order a shipment if a component is at risk of unavailability during the planning horizon. Each supplier offers multiple modes of shipment with a specific cost, quantity and lead time; some suppliers already scheduled deliveries, which is included in the planning of the inbound distribution. Four transport modes are available to deliver components from the supplier to the manufacturing site (Standard Truck **[tr]**, Express Truck **[ces]**, Boat **[bt]**, and cargo plane **[ap]**); each supplier offers different transportation modes, quantities and lead times. Each demand has a due date and a specific quantity of manufacturing orders. We calculate each shipment's average greenhouse gas emissions and associate this quantity with the manufacturing orders. We run the numerical example on three different scenarios; *(s1)* we consider a scenario where the manufacturing site prioritizes the minimization of greenhouse gas emissions, *(s2)* we consider a scenario where the manufacturing site respects the delays while trying to minimize the supply cost of components if that does not conflict with a long delay, *(s3)* Finally, we consider a scenario where the manufacturing site prioritizes customer satisfaction, orders must be delivered on time whenever possible. Our objective is to find the optimal production plan solution with the minimum cost over a planning horizon of fifteen periods, wherein the first ten periods, we expect deliveries from suppliers (*'t1'* → *'t10'*); in this part of the horizon, we don't produce manufacturing orders; manufacturing orders can be produced in the last five periods of the horizon (*'t11'* → *'t15'*). The following tables (I, II) show the model input parameters. The objective is to minimize the overall operational costs that are exemplified in (*) holding cost of components, (*) greenhouse gas emissions cost, (*) lateness cost, (*) supply cost in each scenario; different weighing factors are introduced to each cost to show how the production and inbound supply plans change according to the manufacturing site's priorities.

TABLE I
DEMAND DATA

	D1	D2	D3	D4	D5
Product Type	PA	PB	PB	PA	PB
Quantity	18	15	8	5	6
Due Date	t14	t12	t12	t11	t15

TABLE II
SUPPLIER DATA

		Transport Mode			
Supplier	**Component(s)**	**tr**	**ces**	**bt**	**ap**
(1)	partA	10,25,8,	5,60,4	20,10,8	5,50,2
(2)	partB,	10,30,6	5,60,3	-	-
	partF	10,25,6	5,50,3	-	-
-	partC	1,0,1	-	-	-
(3)	partD	-	-	20,15,12	-
(4)	partE	-	-	10,10,5	5,60,1

Quantity, Cost, Lead time
partC is produced at the manufacturing site and has a lead time of 1

The production plan of orders over the studied horizon in the three scenarios *s1, s2, s3* is presented in table III. The optimization model used Gurobi ® on a PC with an AMD Ryzen 3 PRO 5450U with Radeon Graphics (2.60 GHz). Execution time is under 15 seconds in all iterations, and the instances were generated randomly while adding rules to avoid infeasible instances.

TABLE III
PRODUCTION PLAN

	D1	D2	D3	D4	D5
t11	1,5,4	0,0,5	0,1,0	0,0,1	0,4,0
t12	0,0,5	1,5,2	0,0,0	0,5,0	0,0,3
t13	0,8,5	1,2,2	4,0,0	5,0,0	0,0,3
t14	0,3,0	0,0,2	4,7,4	0,0,3	6,0,1
t15	6,0,0	4,8,4	0,0,4	0,0,0	0,2,2

scenario (*s1*), scenario (*s2*), scenario (*s3*)

The first scenario *s1* represents a setting where the manufacturing site tries to minimize the greenhouse gas emissions related to the inbound supply. To achieve this, we introduced a higher weight to *c4* to have the following objective value ($1 * c1 + 1 * c2 + 1 * c3 + 10 * c4$); the results show that this decision greatly impacts the supply plan (see Fig. 2) we can notice that no cargo plane has been used, Table III shows that the supply plan impact which manufacturing orders to produce at each period, the manufacturing site was able to reduce their greenhouse gas emission by 72% compared to the other scenarios. The second scenario *s2* represents a manufacturing site that tries to reduce its operational costs, notably the inbound supply costs; in this scenario, we have the following objective value ($1*c1+4*c2+5*c3+1*c4$); the results in Fig. 3) show that the manufacturing site used cargo planes to supply parts, this means the holding cost of the components was reduced by changing the production plan and consuming the components faster than in scenario *s1*. The third scenario represents a situation where the manufacturing site prioritizes customer satisfaction, we have the following objective value ($10*c1+1*c2+1*c3+0.1*c4$), and the results of the production plan are different from other scenarios.

We notice some common decisions in the three scenarios, *partC* is always supplied in the same periods because it is a component produced internally in the manufacturing site, and the holding cost of components plays a vital role in supply and production plans. For example, in scenario *s1*, the manufacturing site decided to supply *partA* via a standard truck (tr) at periods *'t5'* and *'t7'*. In scenario *s2*, the manufacturing site decided to supply *partA* via a cargo plane (ap) at periods *'t13'*; to reduce the holding cost of components. In the three scenarios presented, the lateness cost was null; this means that the manufacturing site's policies had no impact on the respect of the due dates; in other instances, we notice some lateness in the manufacturing of orders if we prioritize the reduction of supply costs and greenhouse gas emissions much more than we prioritize the respect of due dates and customer satisfaction. The express truck (ces) was not used because it delivers small quantities and the cost is relatively high; we expect the use of express trucks if the manufacturing site prioritizes the reduction of greenhouse gas emissions and has a higher holding cost of components. The service level $SL1$ is the same in all scenarios (96%). However, the service level per customer order $SL2$ is different in *s3* (94.8%); in *s1* and *s2* (97.7%); the explanation of this is the holding cost of the components since the service level is not part of the objective function. The solution is optimal within the horizon studied. We can consider the solution on different planning horizons to ensure there is no horizon bias. Finally, we expect fewer supply orders using cargo planes if suppliers introduce more flexible supply quantities with the other transport modes.

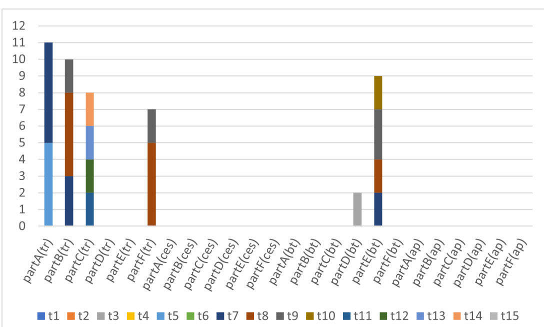

Fig. 2. Supply Plan - scenario *s1*

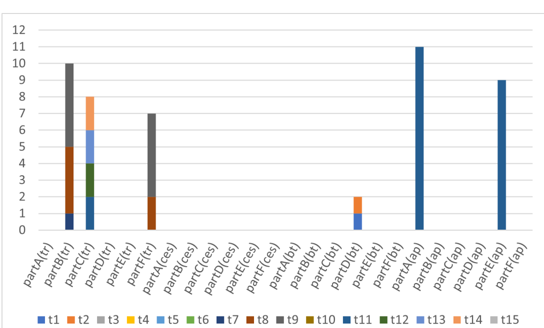

Fig. 3. Supply Plan - scenario *s2* & *s3*

VI. Conclusion

In a dynamic business environment characterized by reduced storage capabilities and minimal waiting times between customer orders and product delivery, the integration of production scheduling and distribution issues has emerged as a pivotal avenue for enhancing overall performance metrics. Companies operate within diverse production environments and utilize various means of delivery, including third-party logistics providers or in-house transportation fleets. Consequently, real-world problems necessitating scheduling and delivery integration exhibit considerable heterogeneity. We presented an optimization model that responds to this need and highlighted how different policies in the manufacturing site can impact the supply and production plans. Even though our third scenario is not close to how the industry currently operates, based on our research, economic (financial) advantages are always prioritized in the automotive industry supply chain compared to greenhouse emissions. We believe that this imbalance of objectives will change in the coming years.

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