Sensitivity and Uncertainty Analysis in a Circulating Fluidized Bed Reactor Modeling

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January 25, 2022

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Abstract

As in many real applications, in the world of fine powders and small particles, depending on the accuracy of the relevant method, there are uncertainties and vagueness in the parameters such as particle size, sphericity, initial solid void fraction, envelope density, etc. In some cases, there are different methods to measure a parameter, such as a particle size that depends on the method (based on length, weight, and volume); the measured values may be significantly different from each other. Therefore, there is no crisp or exactly known parameter in many cases because of the fine powders' inherent uncertain nature. On the other hand, being characteristic of the dynamic systems, physical parameters such as temperature and pressure fluctuate but can be kept in an acceptable range, affecting the main design parameters such as fluid density and dynamic viscosity.

The most traditional tools and methods for simulating, modeling, and reasoning are crisp, deterministic, and precise, but these values are estimated or changing (randomly or stochastically). Several approaches can describe this phenomenon. Moreover, when it comes to uncertainties, mathematical tools are probably the best solutions. With the fuzzy set theory method, linguistic variables or ranges can be converted to mathematical expressions, and consequently, instead of crisp values, these can be applied to the equations. The uncertainty analysis can be more important when the model is susceptible to one parameter. A preliminary sensitivity analysis on a fluidized bed application has shown that the solid void fraction has the highest, and the fluid density has the lowest sensitivity to its operation. The performed uncertain theoretical approach has been validated by CPFD simulation using Barracuda v20.1.0.

Keywords: Fuzzy set theory, Sensitivity analysis, uncertainty analysis, circulating fluidized bed reactor, CPFD simulation

1 Introduction

In general, a solid particle in a fluid behaves in a state of uncertainty. This fact motivates to study the behavior of uncertain phenomena. Most traditional formal modeling, reasoning, and computing tools are crisp, deterministic, and precise. In order to model uncertainty, it is essential to know the uncertainty causes in nature and how it is possible to deal with it. In real-life applications, the system complexity inevitably results in weak models with a high degree of parametric or functional uncertainty. If the controlled system has a multi-valued function or exhibits several modes of behavior during the operation, the problem gets much more complex (Herzallah, 2005).

Generally, gas-solid systems perform pretty in different ways under minor changes of process conditions. For example, changing the velocity from below the minimum fluidization velocity up to a very high velocity, the system experiences many regimes such as the fixed bed, minimum fluidization, smooth, bubbling, slugging, turbulent fluidization, and finally lean phase fluidization with pneumatic transport (Kunii & Levenspiel, 1991). For instance, the hydrodynamics in the fluidized bed is heavily influenced by solid particle properties such as size distribution, sphericity, and voidage. The measurement error, for instance, maybe the most crucial factor for the uncertainty of the particle size distribution (Tinke, 2020).

There are different approaches to categorizing uncertainties in a system. These can be classified into two categories. The first is the uncertainty in a mathematical sense due to the difference between measured, estimated, and actual values, including errors in observations or calculations (Zhu, 2015). The second is the sources of uncertainty, including uncertainty in the particle and fluid physical properties, reaction kinetics (Valkó & Turányi, 2020), reactor temperature, etc.

Traditional and deterministic approaches to a complex system study (such as powder and particulate systems) would not deal with the above uncertainties. Therefore, as seen in Figure 1, the most used uncertainty modeling techniques include probabilistic, possibilistic, and hybrid possibilistic–probabilistic methods, information Gap decision theory (IGDT), and robust optimization (Aien et al., 2016).

These approaches are primarily used to assess the effect of uncertain input parameters on system output parameters. The critical distinction between these

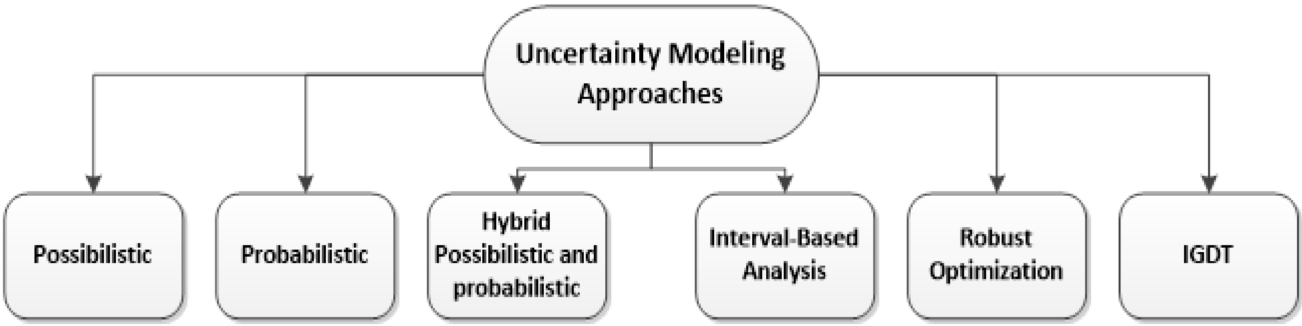

Figure 1. The uncertainty modeling approaches

methods is that they use different ways to describe the ambiguity of input parameters. The following is a short overview of how the above approaches can be used to model uncertainty:

- *Probabilistic approach:* it is assumed that the probability distribution functions of input variables are known. One of the earliest works in stochastic programming was done by (Dantzig, 1955).
- *Possibilistic approach:* a membership function is assigned to model input parameters in this approach (Zadeh, 1999).
- *Hybrid possibilistic–probabilistic approaches:* in this approach, both random and possibilistic parameters are used to handle the uncertain input parameters (Aien et al., 2014; Soroudi & Ehsan, 2011).
- *Information Gap Decision Theory (IGDT):* contrary to probabilistic and possibilistic decision theory, this does not use probability distribution or membership function. Instead, it measures the deviation of differences between parameters and their estimates, but not the probability of outcomes (Ben-Haim, 2001).
- *Robust optimization:* For describing the uncertainty of input parameters, uncertainty sets are used. Obtained decisions are optimal for the worst-case realization of the uncertain parameter within a given set by using this technique (Soyster, 1973).
- *Interval analysis:* The unknown parameters are assumed to take their values from a known interval. It resembles probabilistic simulation with a uniform probability distribution function in several ways (Moore et al., 2009).

Between these causes, the lack of information and measurement errors found in the system can be modeled with the fuzzy set theory, which was first introduced in 1965 (Zadeh, 1965). Reducing the weaknesses of the probability theory, Zadeh introduced the possibility theory (Zadeh, 1999), which naturally complements the fuzzy set theory for handling uncertainty induced by fuzzy and incomplete pieces of information. Possibility theory turns out to be a non-probabilistic view of uncertainty that aims to model states of partial or complete ignorance rather than capture randomness. Using this theory, Dubois and parade (Dubois & Prade, 1983) have studied the ranking of fuzzy numbers considering the possibility and necessity of events. (Goetschel & Voxman, 1986) have introduced a model for ranking fuzzy numbers, which become the primary notion for introducing possibilistic moments by Carlson and Fuller (Carlsson & Fullér, 2001).

On the other hand, the term "sensitivity" describes how our outcomes vary when assumptions in our model are changed. When sensitivity is high, the results fluctuate dramatically when specific assumptions are changed; these assumptions must be extremely well established (Fragoulakis et al., 2015). The Sensitivity Analysis (SA) method is a numerical model that examines how uncertainties in one or more input variables might lead to uncertainties in the output variables (Pichery, 2014). In general, there are two approaches to sensitivity analysis, global and local. The behavior of input parameters on the change of the model output is the focus of global SA, while a local SA looks at sensitivity concerning a single parameter value change. In contrast, a global analysis looks at sensitivity throughout the parameter field (Abedi et al., 2016).

The present study aims to find the parameters that a circulating fluidized bed is sensitive to (using the local SA method) and apply the fuzzy set theory to the mathematical calculations. As numerical examples, the calculated minimum fluidization velocity and cyclone efficiency for the alumina chlorination FBR (Barahmand et al., 2021a) will be compared with the CPFD results. The mathematical approach focuses on Generalized Trapezoidal Fuzzy Numbers (GTrFN) algebraic operations through α-cuts (Zhang et al., 2014) and its application in the CFB.

The present paper describes the fuzzy sets' basic definitions and algebraic operations, properties, and sensitivity analysis. Finally, some numerical examples have adopted the fuzzy model to calculate the minimum fluidization velocity and cyclone efficiency under uncertainty.

2 Fuzzy Set Basics

This section introduces the basic concepts and definitions used in fuzzy sets theory to facilitate future discussions. The notation and concepts introduced by (Carlsson & Fullér, 2001), (Fullér & Majlender, 2003), and (Zimmermann, 1985) are used in this section.

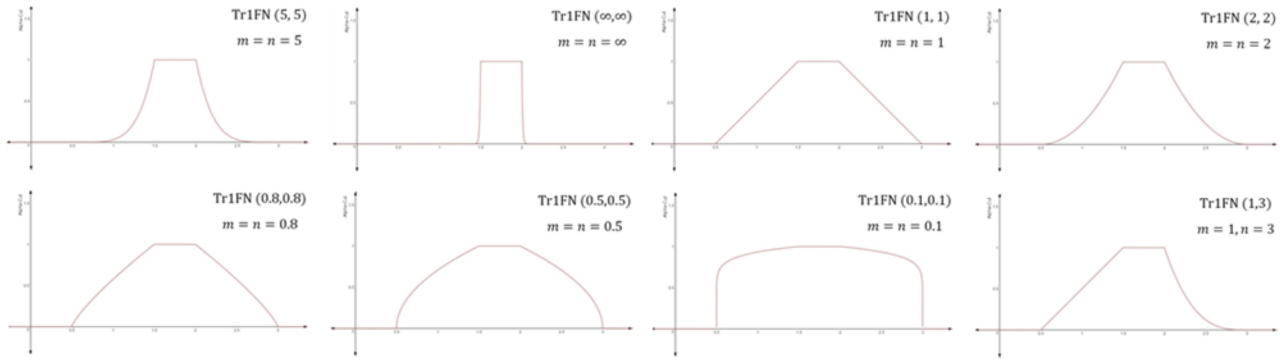

Figure 2 The generalized trapezoidal fuzzy number applying for different orders

2.1 Definitions

Definition 1 fuzzy set A, denoted $\tilde{A}$, is characterized by a Membership Function (MF) $\mu_{\tilde{A}}(x)$, where $x \in X$ (Thavaneswaran et al., 2009).

$$\tilde{A} = \{(x, \mu_A(X)) | x \in A\} \quad , \quad \mu_A(X): X \rightarrow \{0,1\} \qquad (1)$$

$\mu_{\tilde{A}}(x)$ is the degree of membership of X in $\tilde{A}$. The closer the value of $\mu_{\tilde{A}}(x)$ is to 1, the more x belongs to $\tilde{A}$.

Definition 2 Let $\tilde{A}$ be a fuzzy set in X. Then the support of $\tilde{A}$, denoted by $Supp(A)$, is the crisp set given by,

$$Supp(A) = \{x \in X: \mu_{\tilde{A}}(x) > 0\}, \qquad (2)$$

Definition 3 Let $\tilde{A}$ be a fuzzy set in X. The height $h(A)$ of $\tilde{A}$ is defined as,

$$Sh(A) = \sup_{x \in X} \mu_{\tilde{A}}(x) \qquad (3)$$

Definition 4 If $h(A) = 1$, then the fuzzy set $\tilde{A}$ is called a normal fuzzy set.

Definition 5 A Fuzzy Number $\tilde{A}$ is a fuzzy set on the real-line $\mathscr{R}$, which possesses the following properties (Carlsson & Fullér, 2001).

(1) A is a normal, convex fuzzy set on R,

(2) The $A(\alpha)$ is a closed interval for every $\alpha \in (0,1]$,

(3)The membership function is an upper semi-continuous, and

(4) The support of $\tilde{A}$, $S(A) = \{x \in A : \mu_A(X) > 0\}$, is bounded.

Definition 6 As shown in Figure 2, a trapezoidal fuzzy number $\tilde{A} = [a_1, a_2, a_3, a_4]_{(m,n)}$ is defined to be Generalized Trapezoidal Fuzzy Number having orders of m and n (GTrFN or TrFN(m, n)) if the MF is given by,

$$\mu_A(x) = \begin{cases} 0 & x \leq a_1 \\ h(A)\left(\dfrac{x - a_1}{a_2 - a_1}\right)^m & a_1 \leq x \leq a_2 \\ h(A) & a_2 \leq x \leq a_3 \\ h(A)\left(\dfrac{x - a_4}{a_3 - a_4}\right)^n & a_3 \leq x \leq a_4 \\ 0 & x \geq a_4 \end{cases} \qquad (4)$$

Definition 7 An $\alpha - cut$ (interval of confidence) denoted by $A(\alpha)$ is the crisp set of elements x∈$\mathcal{R}$ whose degree of belonging to the fuzzy set $\tilde{A}$ is at least α (Thavaneswaran et al., 2013).

$$A(\alpha) = \{x \in A | \mu_A(X) \geq \alpha \in (0,1]\} = A(\alpha) = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] \qquad (5)$$

Putting membership functions in definition 6 equal to α, and by finding x, the equation (5) will be reached.

$$A(\alpha) = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] = \left[a_1 + (a_2 - a_1)\left(\frac{\alpha}{h(A)}\right)^{\frac{1}{m}}, a_4 + (a_3 - a_4)\left(\frac{\alpha}{h(A)}\right)^{\frac{1}{n}}\right. \qquad (6)$$

Definition 8 Defuzzification (Karnik & Mendel, 2001) of a fuzzy set $\tilde{A}$ with Center of Gravity (CoG) method can be defined as whose domain, $x \in X$, is discretized into N sub-areas, $A_1, A_2, A_3, \ldots, A_N$, is given as,

$$c_{\tilde{A}} = \frac{\sum_{i=1}^{N} x_i A_i}{\sum_{i=1}^{N} A_i} \qquad (7)$$

where, x_i is the CoG in sub-areas.

Definition 9 The f-weighted possibilistic mean value of a fuzzy number ($\tilde{A} = [a_1, a_2, a_3, a_4]_{(m,n)}$) is defined as (Carlsson & Fullér, 2001; Fullér & Majlender, 2003),

$$\bar{M}(A) = \frac{a_1 + a_4}{2} + \frac{(a_2 - a_1)m}{(2m + 1)} + \frac{(a_3 - a_4)n}{(2n + 1)} \qquad (8)$$

3 Sensitivity Analysis

The local sensitivity analysis (Zhou & Lin, 2008) technique defines how an independent variable will impact a specific dependent variable under a given set of assumptions. In this model, the sensitivity of the minimum fluidization velocity to five different parameters (which have uncertainty in nature) has been studied. These parameters are the voidage at the minimum fluidization condition, fluid and solid particles density, average particle diameter, particle sphericity, and gas viscosity.

In the simplified case, with microscopic particles ($Re_{mf} < 20$), the minimum fluidization velocity can be calculated by,

$$u_{mf} = \frac{d_p^2\,(\rho_s - \rho_g)g}{150\,\mu} \cdot \frac{\varepsilon_{mf}^3\,\emptyset_s^2}{1 - \varepsilon_{mf}} \quad (9)$$

where, Re_{mf} is Reynolds number at the minimum fluidization condition, g is the acceleration gravity, ε_{mf} is the voidage at the minimum fluidization condition, ρ_g and ρ_s are the fluid and solid particles density, respectively, d_P is the average particle diameter, μ is the fluid dynamic viscosity and $\emptyset_s$ is the particle sphericity.

Table 1. Sensitivity Analysis of minimum fluidization velocity in a fluidized bed

	Output	Input
	u_{mf}	ε_{mf}
Initial	0.00452	0.4
Secondary	0.01059	0.5
% changed	134%	25%
Sensitivity	134/25 = 5.36	
	u_{mf}	ρ_g
Initial	0.01059	0.93
Secondary	0.01059	2
% changed	0%	115%
Sensitivity	0/115 = 0	
	u_{mf}	d_P
Initial	0.01059	0.000098
Secondary	0.01588	0.000120
% changed	49.9%	22.5%
Sensitivity	49.9/22.5 = 2.22	
	u_{mf}	$\emptyset_s$
Initial	0.00939	0.8
Secondary	0.01188	0.9
% changed	26.5%	12.5
Sensitivity	26.5/12.5 = 2.12	
	u_{mf}	μ
Initial	0.01059	0.000042
Secondary	0.00747	0.000060
% changed	-29.5%	42.8%
Sensitivity	29.5/42.8 = 0.68	

The results in Table 1 show that the highest sensitivity belongs to the ε_{mf}. On the contrary, the model is not sensitive to the ρ_g. The fluid's dynamic viscosity has the second-lowest sensitivity. When it comes to defining a value for each, the value for the voidage must be chosen as accurately as possible because it has the highest sensitivity in the model.

4 Fuzzy Models

4.1 Minimum Fluidization Velocity

Using definition (6), except for solid particle density and acceleration of gravity can be assumed as deterministic parameters, other parameters are considered as a GTrFN. Therefore,

$\tilde{d}_p = (\tilde{d}_{p1}, \tilde{d}_{p2}, \tilde{d}_{p3}, \tilde{d}_{p4}, h(\tilde{d}_p))_{m,n}$

$\tilde{\rho}_g = (\tilde{\rho}_{g1}, \tilde{\rho}_{g2}, \tilde{\rho}_{g3}, \tilde{\rho}_{g4}, h(\tilde{\rho}_g))_{m,n}$

$\tilde{\varepsilon}_{mf} = (\tilde{\varepsilon}_{mf1}, \tilde{\varepsilon}_{mf2}, \tilde{\varepsilon}_{mf3}, \tilde{\varepsilon}_{mf4}, h(\tilde{\varepsilon}_{mf}))_{m,n}$

$\tilde{\mu} = (\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3, \tilde{\mu}_4, h(\tilde{\mu}))_{m,n}$

$\tilde{\emptyset}_s = (\tilde{\emptyset}_{s1}, \tilde{\emptyset}_{s2}, \tilde{\emptyset}_{s3}, \tilde{\emptyset}_{s4}, h(\tilde{\emptyset}_s))_{m,n}$

The fuzzy form of the equation (7) is,

$$\tilde{u}_{mf} = \frac{\tilde{d}_p^2\,(\rho_s - \tilde{\rho}_g)g}{150\,\tilde{\mu}} \cdot \frac{\tilde{\varepsilon}_{mf}^3\,\tilde{\emptyset}_s^2}{1 - \tilde{\varepsilon}_{mf}} \quad (10)$$

Based on the introduced procedure in (Appadoo, 2006) and definition (7), the upper and lower α–cuts of the $\tilde{u}_{mf}$ ($\tilde{u}_{mf} = [\tilde{u}_{mf}^1(\alpha), \tilde{u}_{mf}^2(\alpha)]$), can be written as below:

$$\tilde{u}_{mf}^1(\alpha) = \frac{\tilde{d}_{p1}^2\,(\rho_s - \tilde{\rho}_{g1})g\tilde{\varepsilon}_{mf1}^3\,\tilde{\emptyset}_{s1}^2}{150\,\tilde{\mu}_1(1 - \tilde{\varepsilon}_{mf1})} + \left(\frac{\tilde{d}_{p2}^2\,(\rho_s - \tilde{\rho}_{g2})g\tilde{\varepsilon}_{mf2}^3\,\tilde{\emptyset}_{s2}^2}{150\,\tilde{\mu}_2(1 - \tilde{\varepsilon}_{mf2})} - \frac{\tilde{d}_{p1}^2\,(\rho_s - \tilde{\rho}_{g1})g\tilde{\varepsilon}_{mf1}^3\,\tilde{\emptyset}_{s1}^2}{150\,\tilde{\mu}_1(1 - \tilde{\varepsilon}_{mf1})}\right)\left(\frac{\alpha}{h(\tilde{u}_{mf})}\right)^{\frac{1}{m}} \quad (11)$$

$$\tilde{u}_{mf}^2(\alpha) = \frac{\tilde{d}_{p4}^2\,(\rho_s - \tilde{\rho}_{g4})g\tilde{\varepsilon}_{mf4}^3\,\tilde{\emptyset}_{s4}^2}{150\,\tilde{\mu}_4(1 - \tilde{\varepsilon}_{mf4})} + \left(\frac{\tilde{d}_{p3}^2\,(\rho_s - \tilde{\rho}_{g3})g\tilde{\varepsilon}_{mf3}^3\,\tilde{\emptyset}_{s3}^2}{150\,\tilde{\mu}_3(1 - \tilde{\varepsilon}_{mf3})} - \frac{\tilde{d}_{p4}^2\,(\rho_s - \tilde{\rho}_{g4})g\tilde{\varepsilon}_{mf4}^3\,\tilde{\emptyset}_{s4}^2}{150\,\tilde{\mu}_4(1 - \tilde{\varepsilon}_{mf4})}\right)\left(\frac{\alpha}{h(\tilde{u}_{mf})}\right)^{\frac{1}{n}} \quad (12)$$

As a result, the standard form of the fuzzy minimum fluidization velocity will be:

$$\tilde{u}_{mf} = (\tilde{u}_{mf1}, \tilde{u}_{mf2}, \tilde{u}_{mf3}, \tilde{u}_{mf4}, h(\tilde{u}_{mf}))_{m,n} \quad (13)$$

where, $h(\tilde{u}_{mf})$ can be calculated as the following,

$$h(\tilde{u}_{mf}) = Min\{h(\tilde{d}_p), h(\tilde{\rho}_g), h(\tilde{\varepsilon}_{mf}), h(\tilde{\mu}), h(\tilde{\emptyset}_s)\} \quad (14)$$

$$\mu_{\tilde{u}_{mf}}(x) = \begin{cases} 0 & x \le \tilde{u}_{mf1} \\ h(\tilde{u}_{mf})\left(\dfrac{x - \dfrac{\tilde{d}_{p1}^2\,(\rho_s - \tilde{\rho}_{g1})g\tilde{\varepsilon}_{mf1}^3\,\tilde{\emptyset}_{s1}^2}{150\,\tilde{\mu}_1(1-\tilde{\varepsilon}_{mf1})}}{\dfrac{\tilde{d}_{p2}^2\,(\rho_s - \tilde{\rho}_{g2})g\tilde{\varepsilon}_{mf2}^3\,\tilde{\emptyset}_{s2}^2}{150\,\tilde{\mu}_2(1-\tilde{\varepsilon}_{mf2})} - \dfrac{\tilde{d}_{p1}^2\,(\rho_s - \tilde{\rho}_{g1})g\tilde{\varepsilon}_{mf1}^3\,\tilde{\emptyset}_{s1}^2}{150\,\tilde{\mu}_1(1-\tilde{\varepsilon}_{mf1})}}\right)^m & \tilde{u}_{mf1} \le x \le \tilde{u}_{mf2} \\ h(\tilde{u}_{mf}) & \tilde{u}_{mf2} \le x \le \tilde{u}_{mf3} \\ h(\tilde{u}_{mf})\left(\dfrac{x - \dfrac{\tilde{d}_{p4}^2\,(\rho_s - \tilde{\rho}_{g4})g\tilde{\varepsilon}_{mf4}^3\,\tilde{\emptyset}_{s4}^2}{150\,\tilde{\mu}_4(1-\tilde{\varepsilon}_{mf4})}}{\dfrac{\tilde{d}_{p3}^2\,(\rho_s - \tilde{\rho}_{g3})g\tilde{\varepsilon}_{mf3}^3\,\tilde{\emptyset}_{s3}^2}{150\,\tilde{\mu}_3(1-\tilde{\varepsilon}_{mf3})} - \dfrac{\tilde{d}_{p4}^2\,(\rho_s - \tilde{\rho}_{g4})g\tilde{\varepsilon}_{mf4}^3\,\tilde{\emptyset}_{s4}^2}{150\,\tilde{\mu}_4(1-\tilde{\varepsilon}_{mf4})}}\right)^n & \tilde{u}_{mf3} \le x \le \tilde{u}_{mf4} \\ 0 & x \ge \tilde{u}_{mf4} \end{cases} \quad (15)$$

The membership function of the $\tilde{u}_{mf}$ can be calculated from equation (15).

4.2 Cyclone Efficiency

In general, cyclones are the most common kind of mechanical separator. This basic system has very high efficiency with a low-pressure drop without any moving mechanical components, which are the most favorable advantages. A cyclone is a device that separates solid particles from a fluid by centrifugal force and works simply by the kinetic energy of the incoming mixture (flow stream) and the geometry of the cyclone. Particle (in fluid) velocity and residence time are two main factors in cyclone design (Cooper & Alley, 2010). A typical cyclone scheme is shown in Figure 3.

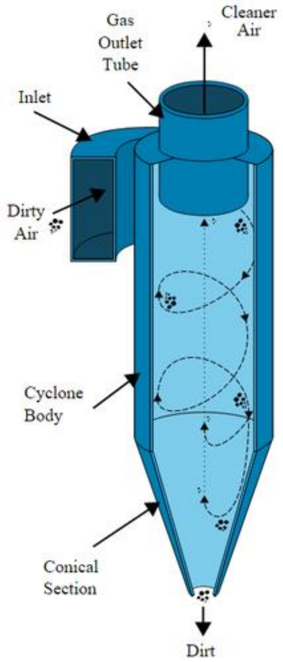

Figure 3. Components of a vertical axis tangential entry cyclone (Afework et al., 2018)

Because of the cyclone's cylindrical form and the tangential entrance of the gas, the gas-solid suspension flows in two concentric vortices around the cyclone. The outer vortex is heading downward, while the central vortex is moving upward. Solids with a higher density than flue gas exit the outer vortex and pass against the wall due to centrifugal force. The comparatively clean gas rises through the inner vortex and leaves through a vertical exit on the cyclone's top (Basu, 2015).

Many parameters affect cyclone efficiency. Table 2 shows the effect of design and process parameters on cyclones' efficiency (Cooper & Alley, 2010). If the parameter increases, the cyclone's efficiency will:

Table 2. Effect of parameters on the cyclone efficiency

Parameter	
Particle size	Increase
Particle density	Increase
Dust loading	Increase*
Inlet gas velocity	Increase*
Cyclone body diameter	Decrease
The ratio of body length to diameter	Increase
The smoothness of cyclone's inner wall	Increase
Gas viscosity	Decrease
Gas density	Decrease
Gas inlet duct area	Decrease
Gas exit pipe diameter	Decrease

*With these parameters, cyclone efficiency can only increase to a certain point and then decrease.

Similarly, as explained in Section 4.1, the uncertainty has been applied to the cyclone calculations based on the

Table 3. Model's deterministic and fuzzy parameters

	Value	Crisp Value	Normal Linear Trapezoidal Fuzzy Number	Weighted P-Mean	Defuzzified (CoG method)
d_p	μm	98	$(20, 75, 115, 190, 1)_{1,1}$	103	102
ρ_s	kg/m^3	3958	-	-	-
ρ_g	kg/m^3	9.29e-01	$(1.04, 9.38e\text{-}01, 9.19e\text{-}01, 8.24e\text{-}01, 1)_{1,1}$	9.32e-01	9.31e-01
ε_{mf}	-	0.4	$(0.35, 0.39, 0.41, 0.42, 1)_{1,1}$	0.39	0.4
g	m/s^2	9.8	-	-	-
μ	kg/ms	4.2e-05	$(3.68e\text{-}05, 4.23e\text{-}05, 4.39e\text{-}05, 5.04e\text{-}05, 1)_{1,1}$	4.3e-05	4.3e-05
$\emptyset_s$	-	0.85	$(0.5, 0.65, 0.75, 0.90, 1)_{1,1}$	0.7	0.7

Lapple method (Cooper & Alley, 2010). As a result, the cyclone efficiency can be calculated by equation 16.

$$\eta = \frac{1}{1+\left(\frac{d_{50}}{d_p}\right)^2} \tag{16}$$

where, η is the cyclone efficiency, d_p is the average particle diameter and d_{50} can be calculated by the equation below.

$$d_{50} = \sqrt{\frac{9\mu W}{2\pi\vartheta_{in}(\rho_p-\rho_g)N_A}} \tag{17}$$

where, ϑ_{in} in the fluid's superficial velocity inlet to the cyclone, W is the cyclone's inlet width and N_A is:

$$N_A = \frac{L_b + 0.5L_c}{H} \tag{18}$$

where, L_b is the cyclone main body height, L_c is the height of the conical part of the cyclone, and H is the height of the inlet of the cyclone. All the parameters in this equation are deterministic. In equations 16 and 17, except N_A, ρ_p and W other parameters can be assumed as the fuzzy number as below,

$\tilde{d}_p = (\tilde{d}_{p1}, \tilde{d}_{p2}, \tilde{d}_{p3}, \tilde{d}_{p4}, h(\tilde{d}_p))_{m,n}$

$\tilde{\rho}_g = (\tilde{\rho}_{g1}, \tilde{\rho}_{g2}, \tilde{\rho}_{g3}, \tilde{\rho}_{g4}, h(\tilde{\rho}_g))_{m,n}$

$\tilde{\vartheta}_{in} = (\tilde{\vartheta}_1, \tilde{\vartheta}_2, \tilde{\vartheta}_3, \tilde{\vartheta}_4, h(\tilde{\vartheta}_{in}))_{m,n}$

$\tilde{\mu} = (\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3, \tilde{\mu}_4, h(\tilde{\mu}))_{m,n}$

In equation 15, defining the fuzzy d_p is sufficient because the value for d_{50} (which all the uncertainties have been considered) will be calculated accordingly.

Equation 16 is an intermediate equation that can be used in the central equation. Therefore, the *possibilistic mean value* of the uncertain parameters can be used to calculate d_{50}. Based on definition 9, equation (17) can be written in the following form:

$$\bar{d}_{50} = \sqrt{\frac{9\bar{\mu} W}{2\pi\bar{\vartheta}_{in}(\rho_p-\bar{\rho}_g)N_A}} \tag{19}$$

where, $\bar{\mu}$, $\bar{\vartheta}_{in}$, and $\bar{\rho}_g$ are the possibilistic mean value for the corresponding parameters.

Now, the fuzzy form of the cyclone's efficiency can be written as,

$$\tilde{\eta} = \frac{1}{1+\left(\frac{\bar{d}_{50}}{\tilde{d}_p}\right)^2} \tag{20}$$

Based on definition 7, the upper and lower α–cuts of the $\tilde{\eta}$ can be written as below:

$$\tilde{\eta} = \frac{1}{1+\left(\frac{\bar{d}_{50}}{\tilde{d}_{p1}}\right)^2} + \left(\frac{1}{1+\left(\frac{\bar{d}_{50}}{\tilde{d}_{p2}}\right)^2} - \frac{1}{1+\left(\frac{\bar{d}_{50}}{\tilde{d}_{p1}}\right)^2}\right)\left(\frac{\alpha}{h(\tilde{\eta})}\right)^{\frac{1}{m}} \tag{21}$$

$$\tilde{\eta} = \frac{1}{1+\left(\frac{\bar{d}_{50}}{\tilde{d}_{p4}}\right)^2} + \left(\frac{1}{1+\left(\frac{\bar{d}_{50}}{\tilde{d}_{p3}}\right)^2} - \frac{1}{1+\left(\frac{\bar{d}_{50}}{\tilde{d}_{p4}}\right)^2}\right)\left(\frac{\alpha}{h(\tilde{\eta})}\right)^{\frac{1}{n}} \tag{22}$$

where, $h(\tilde{\eta})$ can be calculated as the following,

$$h(\tilde{\eta}) = Min\{h(\tilde{d}_p), h(\tilde{\rho}_g), h(\tilde{\vartheta}_{in}), h(\tilde{\mu})\} \tag{23}$$

5 Numerical Examples

5.1 Minimum fluidization Velocity

The following trapezoidal fuzzy parameters are defined for the alumina chlorination in a fluidized bed reactor (Barahmand et al., 2021b). There are different methods to define a fuzzy number. For fuzzy particle diameter, as an example, this can be achieved by fitting a trapezoidal to the size distribution diagram (Figure 4). In this case, the fuzzy particle diameter can be defined as,

$\tilde{d}_p = (20, 75, 115, 190, 1)_{1,1}$

This fuzzy number shows that the particles with a diameter less and more than 25 and 190 microns do not belong to this fuzzy set (the range is between extreme values). The other helpful information given by this fuzzy number is that the particles with a diameter of 85-125 microns 100% belong to this set. To make it more straightforward, as an example, assume a set defined as the black balls. The white, light grey, dark grey, and black balls belong to this set with different belonging degrees. In this case, the belonging degree in the range [0, 1] for these balls is 0, 0.2, 0.8, and 1, respectively.

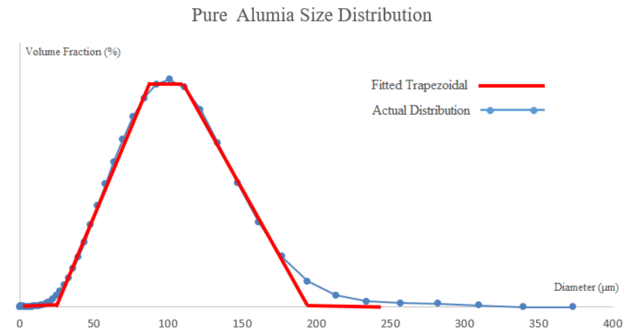

Figure 4. Fitted trapezoidal to the alumina size distribution

Similarly, the other parameters can be defined by operating conditions, results from the dynamic system (fluctuations), experiments, etc.

Table **3** gives the deterministic and fuzzy values used in the numerical example. As an example, for the fluid's density and dynamic viscosity, the interval of the midpoints and endpoints are calculated based on ± 10°C and ± 40°C, respectively.

By applying the data into equations 11 and 12, the minimum fluidization velocity α–cuts can be derived as,

$$\tilde{u}_{mf} = \left[\tilde{u}_{mf}^1(\alpha)\,, \tilde{u}_{mf}^2(\alpha)\,\right]$$
$$= [0.00012 + 0.00256\alpha\,, 0.02133 - 0.014\alpha]$$

To find the interior and endpoints, let $\alpha = 1$ and $\alpha = 0$ in equation (13). As a result, the fuzzy minimum fluidization velocity can be defined as below,

$\tilde{u}_{mf} = (0.00012, 0.00268, 0.00733, 0.02136)$, where the membership function is,

$$\mu_{\tilde{u}_{mf}}(x) = \begin{cases} 0 & x \le 0.00012 \\ \dfrac{x - 0.00012}{0.00256} & 0.00012 \le x \le 0.00268 \\ 1 & 0.00268 \le x \le 0.00733 \\ \dfrac{x - 0.02136}{-0.014} & 0.00733 \le x \le 0.02136 \\ 0 & x \ge 0.02136 \end{cases}$$

Now, based on the above membership and calculated α-cuts (Table 4) and the graphical fuzzy minimum fluidization velocity is presented in Figure 5.

Table 4. $\tilde{u}_{mf}$ α-cuts with linear membership functions

α	0.00	0.20	0.40	0.60	0.80	1.00
$\tilde{u}_{mf}^1$	**0.012**	0.063	0.114	0.166	0.20	**0.27**
α	0.00	0.20	0.40	0.60	0.80	1.00
$\tilde{u}_{mf}^2$	**2.14**	1.72	1.43	1.29	1.01	**0.73**

Considering all uncertain and certain parameters, the calculated fuzzy minimum fluidization velocity is given in Figure 4.

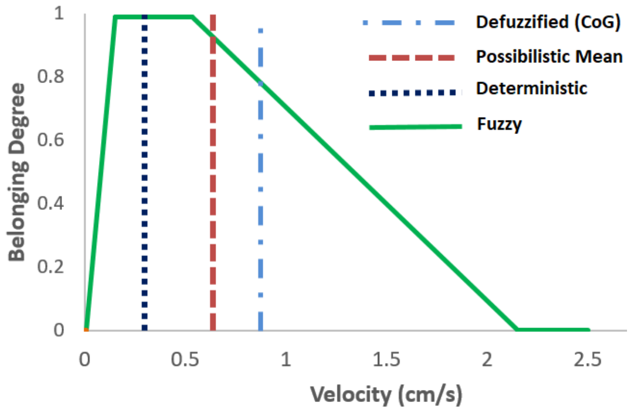

Figure 5. Calculated fuzzy minimum fluidization velocity with linear membership functions.

The results show that the minimum fluidization velocity without considering uncertainty has been calculated as 0.32 cm/s, ideally in the range with the highest belonging degree in the fuzzy number. The fuzzy minimum fluidization velocity gives more information. This analysis illustrates that considering all the defined uncertainties, the minimum fluidization velocity will not be more than 2.14 cm/s and not drop below 0.012 cm/s, but the velocities in the range [0.27, 0.73] cm/s have the highest belonging degree to this set. As seen in Figure 5, the average deterministic value of this fuzzy number (defuzzified based on the center of gravity method and possibilistic mean). Instead of the deterministic calculated value, these values can be used in further reactor design calculations, representing the model's uncertainty.

5.2 Cyclone Efficiency

5.2.1 Base model

To study the performance of the cyclone in a specific operating condition, a CPFD simulation has been done to study the performance of the cyclone in a specific operating condition. The cyclone diameter has been chosen 0.5 m, and all other dimensions can be calculated accordingly. The model has been simulated under the following operating condition, as shown in Table 5.

Figure 6 shows a snapshot of the cyclone simulation in an isothermal condition. As it is clear, most of the particles leave the system from the bottom. Therefore, the average cyclone efficiency can be calculated by dividing the average particle mass flow rates between the bottom and the inlet.

Table 5. Cyclone's operating conditions used for the simulations

Number of cells in setup grid:	500000
Fluid superficial velocity (inlet)	36.5 m/s
Particle duty in:	0.3 kg/s
Temperature:	973.15 K
Outlet pressure:	1.5 bars
Average particle diameter:	20 microns
Fluid density:	1.3318 kg/m^3
Fluid dynamic viscosity:	0.0000287893 pa.s
Cyclone type:	High-Efficiency
Nominal efficiency:	99%
Particle density:	2100 kg/m^3

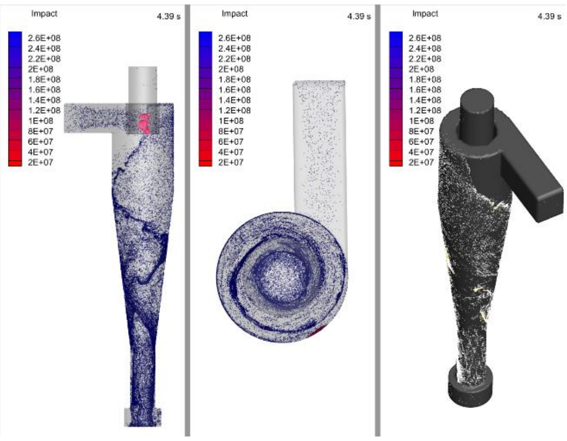

Figure 6. Particle distribution inside the cyclone

5.2.2 Uncertainty in Theoretical Approach

As discussed in Section 4.2, for the first step, the fuzzy parameters should be defined. The particle diameter and superficial velocity inlet to the cyclone are assumed as fuzzy numbers with linear membership functions (*m* and *n* equal to 1). Other parameters are kept the same as Table 5.

$\tilde{d}_p = (10, 18, 20, 22, 1)$

$\tilde{\vartheta}_{in} = (35.64, 36.42, 36.92, 37.34, 1)$

Following the procedure in Section 4.2, in the second step, using definition 9, $\bar{d}_{50}$ can be calculated by using possibilistic mean values in equation 19. From equations 21 and 22, the fuzzy efficiency of the cyclone can be calculated as Figure 7.

5.2.3 Uncertainty in CPFD Model

In the base model, the particle duty entered into the system has been set to 0.3 kg/s. To study the uncertainty using Barracuda® as the best alternative, the particle duty inlet to the cyclone has been chosen because all other uncertain parameters directly or indirectly affect the particle mass concentration.

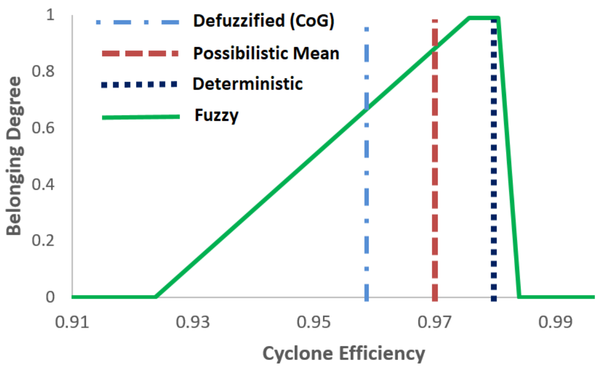

Figure 7. Fuzzy cyclone efficiency

According to Table 6, by increasing the particle duty, the cyclone's efficiency will increase, and after a certain point, it will start to drop. By investigating this with the CPFD simulation, the following results have been observed (Table 6).

Table 6. Sensitivity of Cyclone efficiency to particle concentrations inlet to the cyclone

Particle duty (kg/s)	Particle Escape (kg/s)	Efficiency (%)
0.05	0.002676	94.6
0.1	0.005447	94.6
0.2	0.007239	96.4
0.3	0.008284	97.2
0.4	0.011694	97.1

The calculated fuzzy efficiency is in the overall range of 92.3-98.3%. On the other hand, the CPFD simulation shows the efficiency in the range 94.6-97.1%. Using the parameters in Table 5 and applying equation 16, the theoretical efficiency can be calculated at 98%. Figure 7 clearly shows these calculated efficiency ranges with the highest belonging degree (97.5-98%). On the other hand, there is more information about the efficiency of the system. Considering all the defined uncertainties, the possibility of having efficiency lower than 92.3% and higher than 98.3% is very low, and the efficiency will be in the range of 92.3-98.3%. This range covers the range resulted from CPFD simulation.

6 Conclusion

Solid particles and fine powders in many industrial systems behave in a state of uncertainty. In a circulating fluidized bed, specifically, both sources of uncertainty are available. These sources are the uncertainty in a mathematical sense due to the difference between measured, estimated, and actual values, including errors in observations or calculations, and the uncertainty in particle and fluid physical properties, reaction kinetics, reactor temperature, etc.

The fuzzy set theory is one of the robust tools which can model these uncertainties mathematically. Moreover, applying generalized trapezoidal fuzzy sets to fluidized bed calculation gives designers and analysts a more dependable tool to analyze the uncertainty. As can be seen in the result, the fuzzy model is efficient and valuable, and without introducing this method, it would not be possible to consider this genuine uncertainty.

Overall, it is pretty clear that except for engineering, this fuzzy modeling method has applications in most branches of science and life, such as biomedical sciences, finance, social sciences, etc. Furthermore, future research could extend our model by type-2 incorporating different heights for fuzzy inputs.

References

V. Abedi, R. Hontecillas, A. Carbo, C. Philipson, S. Hoops, and J. Bassaganya-Riera. Chapter 8 - Multiscale Modeling: Concepts, Technologies, and Use Cases in Immunology. In J. Bassaganya-Riera (Ed.), *Computational Immunology* pages 145–173. Academic Press, 2016. doi:10.1016/B978-0-12-803697-6.00008-4

B. Afework, J. Hanania, K. Stenhouse, and J. Donev. *Cyclone separator—Energy Education, 2018*.

M. Aien, A. Hajebrahimi, and M. Fotuhi-Firuzabad. A comprehensive review on uncertainty modeling techniques in power system studies. *Renewable and Sustainable Energy Reviews*, *57*, 1077–1089, 2016. doi:10.1016/j.rser.2015.12.070

M. Aien, M. Rashidinejad, and M. Fotuhi-Firuzabad. On possibilistic and probabilistic uncertainty assessment of power flow problem: A review and a new approach. *Renewable and Sustainable Energy Reviews*, *37*, 883–895, 2014. doi:10.1016/j.rser.2014.05.063

S. S. Appadoo. *Pricing financial derivatives with fuzzy algebraic models: A theoretical and computational approach*, Ph.D. thesis, University of Manitoba, 2006.

Z. Barahmand, C. Jayarathna, and C. Ratnayake. CPFD simulations on a chlorination fluidized bed reactor for aluminum production: An optimization study. *In Proceedings - 1st SIMS EUROSIM Conference on Modelling and Simulation*, Finland, 2021a.

Z. Barahmand, C. Jayarathna, and C. Ratnayake. The effect of alumina impurities on chlorination in a fluidized bed reactor: A CPFD study. *In Proceedings - 1st SIMS EUROSIM Conference on Modelling and Simulation*, Finland, 2021b.

P. Basu. *Circulating Fluidized Bed Boilers: Design, Operation and Maintenance*. Springer International Publishing, 2015. doi:10.1007/978-3-319-06173-3

Y. Ben-Haim. *Information Gap Decision Theory: Decisions under Severe Uncertainty* (1st edition). Academic Press, 2001.

C. Carlsson and R. Fullér. On possibilistic mean value and variance of fuzzy numbers. *Fuzzy Sets and Systems*, *122*(2), 315–326, 2001. doi:10.1016/S0165-0114(00)00043-9

C. D. Cooper and F. C. Alley. *Air Pollution Control: A Design Approach, Fourth Edition*. Waveland Press, 2010.

G. B. Dantzig. Linear Programming under Uncertainty. *Management Science*, *1*(3–4), 197–206, 1955. doi: 10.1287/mnsc.1.3-4.197

D. Dubois and H. Prade. Ranking fuzzy numbers in the setting of possibility theory. *Information Sciences*, *30*(3), 183–224, 1983. doi:10.1016/0020-0255(83)90025-7

V. Fragoulakis, C. Mitropoulou, M. S. Williams, & G. P. Patrinos. Chapter 5—Advanced Methodological Aspects in the Economic Evaluation. In V. Fragoulakis, C. Mitropoulou, M. S. Williams, & G. P. Patrinos (Eds.), *Economic Evaluation in Genomic Medicine* (pp. 65–96). Academic Press, 2015. doi:10.1016/B978-0-12-801497-4.00005-9

R. Fullér and P. Majlender. On weighted possibilistic mean and variance of fuzzy numbers. *Fuzzy Sets and Systems*, 136, 363–374, 2003. doi:10.1016/S0165-0114(02)00216-6

R. Goetschel and W. Voxman. Elementary fuzzy calculus. *Fuzzy Sets and Systems*, *18*(1), 31–43, 1986. doi:10.1016/0165-0114(86)90026-6

Herzallah, R. (2005). Incertainty in control problems: A survey. *IFAC Proceedings Volumes*, *38*(1), 82–90. https://doi.org/10.3182/20050703-6-CZ-1902.01092

N. N. Karnikand J. M. Mendel. Centroid of a type-2 fuzzy set. *Information Sciences*, *132*(1), 195–220, 2001. doi:10.1016/S0020-0255(01)00069-X

D. Kunii and O. Levenspiel. *Fluidization Engineering*. Butterworth-Heinemann, 1991.

R. E. Moore, R.B. Kearfott, and M. J. Cloud. *Introduction to Interval Analysis* (1st edition). Society for Industrial and Applied Mathematics, 2009.

C. Pichery. Sensitivity Analysis. In P. Wexler (Ed.), *Encyclopedia of Toxicology (third edition)*, pages 236–237. Academic Press, 2014. doi:10.1016/B978-0-12-386454-3.00431-0

A. Soroudi and M. Ehsan. A possibilistic-probabilistic tool for evaluating the impact of stochastic renewable and controllable power generation on energy losses in distribution networks: A case study. *Renewable and Sustainable Energy Reviews*, *15*(1), 794–800, 2011.

A. L. Soyster. Convex Programming with Set-Inclusive Constraints and Applications to Inexact Linear

Programming. *Operations Research*, *21*(5), pages 1154–1157, 1973.

A. Thavaneswaran, S. S. Appadoo, and J. Frank, J. Binary option pricing using fuzzy numbers. *Applied Mathematics Letters*, *26*(1), 65–72, 2013. doi: 10.1016/j.aml.2012.03.034

A. Thavaneswaran, S. S. Appadoo, and A. Paseka. Weighted possibilistic moments of fuzzy numbers with applications to GARCH modeling and option pricing. *Mathematical and Computer Modelling*, *49*(1), 352–368, 2009. doi: 10.1016/j.mcm.2008.07.035

A. Tinke. *Minimum Sample Size and Measurement Uncertainty in the Particle Size Distribution Analysis of Powders, 2020*. doi: 10.13140/RG.2.2.12086.45127

É. Valkó and T. Turányi. *Uncertainty Quantification of Chemical Kinetic Reaction Rate Coefficients*, pages 35–44, 2020. doi:10.1007/978-3-030-50388-8_3

L. A. Zadeh. Fuzzy sets. *Information and Control*, *8*(3), 338–353, 1965. doi: 10.1016/S0019-9958(65)90241-X

L. A. Zadeh. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, *100*, 9–34, 1999. doi: 10.1016/S0165-0114(99)80004-9

H. Zhang, L. Shu, and S. Liao. Generalized Trapezoidal Fuzzy Soft Set and Its Application in Medical Diagnosis. *Journal of Applied Mathematics*, 2014. doi: 10.1155/2014/312069

X. Zhou, and H. Lin. Local Sensitivity Analysis. In S. Shekhar & H. Xiong (Eds.), *Encyclopedia of GIS* (pp. 616–616). Springer US, 2008. doi: 10.1007/978-0-387-35973-1_703

J. Zhu. *Optimization of Power System Operation*. Wiley-IEEE Press, 2015.

H. J. Zimmermann. Fuzzy Sets—Basic Definitions. In H.-J. Zimmermann (Ed.), *Fuzzy Set Theory—And Its Applications*, pages 11–23. Springer Netherlands, 1985. doi: 10.1007/978-94-015-7153-1_2