Studying Propagtion of Nonstationgry Dynamical Waves in Inhomogeneous Layered Media

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Abstract: Analytical methodology for solving boundary value problems for two-layered inhomogeneous viscoelatic semi-spaces is developed and exposure of external in influences on medium stability on the constructed mathematical models, is studied. The form of nonstationary surface phenomena corresponding to force boundary conditions is determined. It is obtained that the rate of attenuation of wave that are not attenuated in an elastic layered medium, turns out to be less than the rate of attenuation of waves in areas of non-transmission depending on viscoelastic properties of materials.

Keywords: inhomogeneity, lamination, viscoelastic, Laplace operator, original, image, iterated kernels.

I. INTRODUCTION

Recently, considerable attention is paid to the study of the regularities of interaction of wave processes of different medium. This is related with the fact that many problems of nature are connected with the solution of the problems of seismology and seismic stability of engineering systems, that leads to studing wave processes allowing for inhomogeneity, liminarlity and rheological properties of the medium. The are a lot of waves devoted to this problem [2, 3].

II. PROBLEM STATEMENT

Assume that in the rectangular coordinate system OXYZ at the moment $t = 0$, dynamical load $\sigma_1 = g(t)$ is applied to the surface of a two-layerd inhomogeneous layer. There the study of wave state of the medium is reduced to the solution of the equation of motion [4].

$$\frac{\partial \sigma_i(y,t)}{\partial x} = \rho_i \frac{\partial^2 v_i(x,t)}{\partial t^2} \qquad (1)$$

under the following boundary conditions:

$$\sigma_1 = g(t) \; for \; y = 0 \qquad (2)$$

$$V_2(y,t) = 0 \; for \; y = l_1 + l_2 \qquad (3)$$

The boundary conditions are zero

$$V_i(y,t) = \frac{\partial v_i(y,t)}{\partial t} = 0 \; for \; t = 0 \qquad (4)$$

The contact conditions are accepted in the form:

$$\sigma_1(y,t) = \sigma_2(y,t),$$

$$V_1(y,t) = V_2(y,t) \text{ при } y = l_1 \qquad (5)$$

We determine the defining relations and inhomogeneity of the medium by the following formulas :

$$\sigma_1(y,t) = \int_0^t \Big[R^{(1)}(t-\tau,y) +$$

$$+\frac{2}{3}R^{(i)}(t-\tau,y)\Big]\, d\left(\frac{\partial v_i(y,t)}{\partial y}\right) \qquad (6)$$

$$\rho_i(y) = \rho_i^0(1+by)^m \quad R_i(y,t) = R^i(t)(1+by)^m \; (7)$$

where $i = 1{,}2; j = 1{,}2$; $\sigma_i(y,t)$ is a stress, $\rho_i^0 - const$, is medium density, $V(y,t)$ is displacement, l_1 and l_2 are layer thicknesses, b and m are constants, $R_i^{(1)}(y,t)$ and $R_i^{(2)}(y,t)$ are volume and shear relaxation functions, $g(t)$ is the given external load.

Note that for $i = 1$ all the relations belong to the first layer, for $i = 2$,to the second layer. As can be seen? the solution of the problem is led to the solution of equation (2) under the conditions (2)-(7).

Applying the Laplace transform in time t to the equation (1) allowing for (4), (6) and (7) we obtain:

$$\frac{d^2\bar{V}_i(z,p)}{\partial z^2} + \frac{k}{z}\frac{d\bar{V}_i(z,p)}{\partial z} - \frac{R_i^2}{a^2}\bar{V}(z,p) = 0 \qquad (8)$$

where: $z = 1 + ay$, $\beta_i^2(p) = \frac{p^2}{C_i^2(p)}$; $C_i^2(p) = \frac{pR_i^{(1)}(p)+\frac{2}{3}p\bar{R}_i^{(1)}(p)}{\rho_i^0}$

p is a parameter of Laplace , $\bar{V}_i(z,p)$ is the image of the function $\bar{V}_i(y,t)$, $\bar{R}_i^{(1)}(p)$ and $R_i^2(p)$ are the images of the functions $R_i^{(1)}(t)$, and $R_i^{(2)}(t)$ respectively. The solution of the equation (8), satisfying conditions (2), (3) and (5) has the form:

$$\bar{V}_1(z,p) = \frac{z^y}{\Delta}\left[\Delta_1 K_\nu\left(\frac{\beta_1 z}{a}\right) + \Delta_2 I_\nu\left(\frac{\beta_1 z}{a}\right)\right]$$

$$\bar{V}_2(z,p) = \frac{z^y}{\Delta}\left[\Delta_3 K_\nu\left(\frac{\beta_2 z}{a}\right) + \Delta_4 I_\nu\left(\frac{\beta_2 z}{a}\right)\right] \qquad (9)$$

where

$$\Delta= \left[I_{\nu-1}\left(\frac{\beta_1}{a}\right)K_{\nu-1}\left(\frac{\beta_1 z_1}{a}\right)-K_{\nu-1}\left(\frac{\beta_1}{a}\right)I_{\nu-1}\left(\frac{\beta_1 z_1}{a}\right)\right]\times$$

$$\times\left[I_\nu\left(\frac{\beta_2 z_2}{a}\right)K_\nu\left(\frac{\beta_2 z_1}{a}\right)-K_\nu\left(\frac{\beta_2 z_2}{a}\right)I_\nu\left(\frac{\beta_2 z_1}{a}\right)\right]-$$

$$-\alpha\left[K_{\nu-1}\left(\frac{\beta_1}{a}\right)I_n\left(\frac{\beta_1 z_1}{a}\right)+I_{\nu-1}\left(\frac{\beta_1}{a}\right)K_\nu\left(\frac{\beta_1 z_1}{a}\right)\right]\times$$

$$\times\left[I_\nu\left(\frac{\beta_2 z_2}{a}\right)K_{\nu-1}\left(\frac{\beta_2 z_1}{a}\right)+K_\nu\left(\frac{\beta_2 z_2}{a}\right)I_{\nu-1}\left(\frac{\beta_2 z_1}{a}\right)\right];$$

$$\Delta_1= \frac{\bar{g}(p)}{p\bar{R}_1^{(1)}+\frac{2}{3}p\bar{R}_2^{(1)}}\cdot\frac{1}{\beta_1}\left\{\alpha I_\nu\left(\frac{\beta_1 z_1}{a}\right)\times\right.$$

$$\times\left[K_{\nu-1}\left(\frac{\beta_2 z_1}{a}\right)I_\nu\left(\frac{\beta_2 z_2}{a}\right)+K_\nu\left(\frac{\beta_2 z_2}{a}\right)I_{\nu-1}\left(\frac{\beta_2 z_1}{a}\right)\right]+$$

$$+I_{\nu-1}\left(\frac{\beta_1 z_1}{a}\right)\left[I_\nu\left(\frac{\beta_2 z_2}{a}\right)K_\nu\left(\frac{\beta_2 z_1}{a}\right)-\right.$$

$$\left.\left.-K_\nu\left(\frac{\beta_2 z_2}{a}\right)I_\nu\left(\frac{\beta_2 z_1}{a}\right)\right]\right\}$$

$$\Delta_2= \frac{\bar{g}(p)}{p\bar{R}_1^{(1)}+\frac{2}{3}p\bar{R}_2^{(1)}}\cdot\frac{1}{\beta_1}\left\{K_{\nu-1}\left(\frac{\beta_1 z_1}{a}\right)\times\right.$$

$$\times\left[I_\nu\left(\frac{\beta_2 z_2}{a}\right)K_\nu\left(\frac{\beta_2 z_1}{a}\right)-I_\nu\left(\frac{\beta_2 z_1}{a}\right)K_{\nu-1}\left(\frac{\beta_2 z_2}{a}\right)\right]-$$

$$-\alpha K_\nu\left(\frac{\beta_1 z_1}{a}\right)\left[K_\nu\left(\frac{\beta_2 z_2}{a}\right)I_{\nu-1}\left(\frac{\beta_2 z_1}{a}\right)+\right.$$

$$\left.\left.+I_\nu\left(\frac{\beta_2 z_1}{a}\right)K_{\nu-1}\left(\frac{\beta_2 z_1}{a}\right)\right]\right\}$$

$$\Delta_3= \frac{\bar{g}(p)}{p\bar{R}_1^{(1)}+\frac{2}{3}p\bar{R}_2^{(1)}}\cdot\frac{1}{\beta_1}I_\nu\left(\frac{\beta_2 z_2}{a}\right)\left[K_\nu\left(\frac{\beta_1 z_1}{a}\right)\times\right.$$

$$\left.\times I_{\nu-1}\left(\frac{\beta_1 z_1}{a}\right)+I_\nu\left(\frac{\beta_1 z_1}{a}\right)K_{\nu-1}\left(\frac{\beta_1 z_1}{a}\right)\right]$$

$$\Delta_4= \frac{\bar{g}(p)}{p\bar{R}_1^{(1)}+\frac{2}{3}p\bar{R}_2^{(1)}}\cdot\frac{1}{\beta_1}K_\nu\left(\frac{\beta_2 z_2}{a}\right)\left[I_\nu\left(\frac{\beta_1 z_1}{a}\right)\times\right.$$

$$\left.\times K_{\nu-1}\left(\frac{\beta_1 z_1}{a}\right)+K_\nu\left(\frac{\beta_1 z_1}{a}\right)I_{\nu-1}\left(\frac{\beta_1 z_1}{a}\right)\right]$$

where $I_\nu(z)$ and $K_\nu(z)$ are cylindrical Bessel functions [1, 4].

$$\alpha=\frac{p\bar{R}_1^{(2)}+\frac{2}{3}p\bar{R}_2^{(2)}}{p\bar{R}_1^{(1)}+\frac{2}{3}p\bar{R}_2^{(1)}}\cdot\frac{\beta_2}{\beta_1};\ z_1=1+al_1;\ z_2=1+al_2$$

By means of expansion of cylindrical functions, for large values of arguments we obtain : [1]

$$\bar{V}_1(z,p)=\frac{\bar{g}(p)}{p\rho_1^0 C_1}\sqrt{\frac{R_1^{(1)}(0)}{p\bar{R}_1^{(1)}}}\sum_{n=0}^{\infty}\theta^n\left[\exp\left(-\frac{p}{C_1}\frac{z+2nl_1}{a}\times\right.\right.$$

$$\left.\left.\times\sqrt{\frac{R_1^{(1)}(0)}{p\bar{R}_1^{(1)}}}\right)+\theta\exp\left(-\frac{p}{C_1}\frac{2(n+1)-z}{a}\sqrt{\frac{R_1^{(1)}(0)}{p\bar{R}_1^{(1)}}}\right)\right]$$

$$\bar{V}_2(z,p)=\frac{a\bar{V}_1(l_1,p)}{2\gamma p\sqrt{z_1 z_2}}\sqrt{\frac{R^{(2)}(0)}{pR^{(2)}(p)}}\left[\exp\left(-\frac{p}{C_2}\frac{z-z_2}{a}\times\right.\right.$$

$$\left.\left.\times\sqrt{\frac{R^{(2)}(0)}{p\bar{R}^{(2)}}}\right)-\exp\left(-\frac{p}{C_2}\frac{z_2-z}{a}\sqrt{\frac{R^{(2)}(0)}{pR^{(2)}(p)}}\right)\right] \qquad (10)$$

were $C_i=\sqrt{\frac{R_1^{(i)}(0)+\frac{2}{3}R^{(i)}(0)}{\rho_i^0}}$; $\theta=\frac{\beta_2\gamma z_1+(\gamma y-1)a\mu}{\beta_2\gamma z_1-(\gamma y-1)a\mu}$;

$$\gamma=\frac{z_2-z_1}{z_1};\ \frac{p\bar{R}_1^{(1)}}{pR^{(i)}}=\mu=const.$$

Hence it is seen that the solution of the stated problem is reduced to calculating the inverse Laplace transformation of the function of the form:

$$\bar{\varphi}(z,p)=-\frac{\bar{g}(p)}{p}\sqrt{\frac{R(0)}{p\bar{R}(p)}}exp\left(-\frac{p}{C}\frac{r}{a}\sqrt{\frac{R(0)}{p\bar{R}}}\right)$$

It is known that $p\bar{R}=R_0(i-\varepsilon\bar{\Gamma})$. Taking this into account, representing the exponential function in the form of a Fourier integral, we obtain:

$$\bar{\varphi}(z,p)=-\frac{2\bar{g}(p)}{p\sqrt{1}}\int_0^\infty\frac{cos\left(\frac{\lambda z}{C}\right)d\lambda}{p^2+\lambda^2-\varepsilon\lambda^2\bar{\Gamma}(p)}$$

Taking into account $\left|\frac{\varepsilon\lambda^2\bar{\Gamma}(p)}{p^2+\lambda^2}\right|<1$ we expand the integrand function in series:

$$\varphi(z,p)=-\frac{2\bar{g}(p)}{\pi p}\int_0^\infty\left[\frac{1}{p^2+\lambda^2}+\frac{\varepsilon\lambda^2\bar{\Gamma}(p)}{(p^2+\lambda^2)^2}+\cdots+\right.$$

$$\left.+\frac{(\varepsilon\lambda^2\bar{\Gamma}(p))^m}{(p^2+\lambda^2)^{m+1}}\right]cos\left(\frac{\lambda z}{C}\right)d\lambda$$

Calculating the integrals, we obtain:

$$\varphi(z,p)=-\frac{\bar{g}(p)}{p}e^{-\frac{pz}{C}}\left[\frac{1}{p}+\frac{1}{2^2}\left(\frac{z}{C}\right)\frac{\varepsilon\bar{\Gamma}}{p^2}+\cdots+\right.$$

$$\left.+\frac{p^{-(m+1)}}{2^m m!}\left(\frac{z}{C}\right)^m\left(\varepsilon\bar{\Gamma}(p)\right)^m+\cdots\right]$$

Passing to the space of originals, we find:

$$\varphi(z,p)=-\int_0^t g(\tau)d\tau*\left[H\left(t-\frac{z}{C}\right)+\frac{1}{2^2}\left(\frac{z}{C}\right)\varepsilon\Gamma(\mathrm{t})+\cdots+\right.$$

$$\frac{1}{2^m m!}\varepsilon^m \Gamma^{(m)}(\mathrm{t}) + \cdots \Big]$$

where $\Gamma_1(t) = \Gamma(t)$, $\Gamma^{(2)}(t) = \int_0^t \Gamma(t-\tau)\Gamma_1(\tau)d\tau, \ldots$, $\Gamma^{(m)}(t) = \int_0^t \Gamma(t-\tau)\Gamma^{m-1}(\tau)d\tau$

then the solution is determined by the formulas :

$$V_1(z,t) = -\frac{1}{\rho_1^0 C_1}\int_0^t g(\tau)d\tau * \sum_{n=1}^{\infty}[\varphi(z+2nl_1,t) +$$

$$+\varphi((2(n+1)-z;t)]\theta^n$$

$$V_2(z,t) = \frac{a}{2\gamma\sqrt{z_1 z_2}}\int_0^t V_1(l,t)\,[\varphi(z-z_2,t) - \varphi(z_2-z,t)]$$

III. Conclusion

1. A problem on propagation of nonstationary waves in inhomogeneous viscoelastic laminated semi-spaces for arbitrary hereditary functions at low viscosity was solved by the method of Laplace integral transforms.

2. The influence of viscosity and homogeneity of materials on wave propagation when the density and relaxation function depend on coordinates directed into the subspace was studied.

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