De Rham Cohomology for Compact Kahler Manifolds

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Abstract:

De Rham Cohomology is shown for compact Kahler manifolds considering the Hodge theory, Kähler potential and ddbar lemma.

Keywords: Kähler manifolds, Hodge Theory, ddbar Lemma.

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Introduction:

The ddbar lemma is a powerful tool whose bidegree form is essential in incorporating the De Rham Cohomology that in turn expresses the compact Kahler manifold provided certain conditions are met. Here in this paper the exact and harmonic form is taken along with the De Rham Cohomology class to compute the Kahler potential and ddbar lemma whose consequence is the Hodge theory.

Methodology:

Let there be an external mapping of sections for the exterior derivative Δ for the mapping of the projections that in general turned out to be the Dolbeault operators that in essence be the,

$$\Delta:\ \Omega^r \longrightarrow \Omega^{r+1}$$

Where for the De Rham Cohomology one can describe the 4 forms as ϵ, ϵ_1, ϵ_2, ϵ_3 for a equivalence class of closed forms $[\epsilon]$ having the representation,

$$\epsilon_1 = \epsilon_2 + \epsilon_3$$

Where for $\epsilon_1 \cong [\epsilon]$, one can get the harmonic form $\Delta\epsilon_3 = 0$ for the exact form ϵ_2. Thus, if we take a constant sheaf on $\overline{\mathcal{R}}$ for a smooth manifold S where the De Rham Cohomology showed that for any map parameterized by J one can easily make the mapping [1-8, 14-18],

$$J : H^p_{dR}(S) \longrightarrow H^p\left(S,\overline{\mathcal{R}}\right) \ni \begin{cases} [\epsilon] \in H^p_{dR}(S) \\ \forall J_{\epsilon_1} \in Hom\ H_p\left(S,\overline{\mathcal{R}}\right) \equiv H^p\left(S,\overline{\mathcal{R}}\right) \end{cases}$$

This proves two identities;

1. One can have a natural isomorphism for the sheaf cohomology of $\overline{\mathcal{R}}$ in a way as to show,

$$H^*_{dR}(S) \cong H^*\left(S,\overline{\mathcal{R}}\right)$$

$\ni\mathcal{R}$ represents a Abelian group.

2. One gets the isomorphism between the singular cohomology and de Rham cohomology such that for ant set $\mathcal{G}$ and a trivial parameter classification or class $[G]$ there exists the relation with ϵ_1 in the way,

$$T \approx [G] \longrightarrow \sum\nolimits_G \epsilon_1$$

$$\forall \begin{cases} T \in [G] \\ T \in H_p(S) \end{cases}$$

Thus, one can take the exact form ϵ_2 and differentiating it with i, j, k for a grouping of $\langle |i|, |j| \rangle$ the resultant factor provides the relation to $\Delta:\ \Omega^r \longrightarrow \Omega^{r+1}$ such that in the case of ϵ_2 one of the most important aspects of Hodge theory can be found giving the wedge form,

$$\Omega^{p,q} \ni \epsilon_2 \cong \sum_{p,q} f_{ij}\Delta z^i \wedge \Delta\bar{z}^j$$

$\ni in\ \langle |i|, |j| \rangle;\ |i| = p, |j| = q$ for $\Omega^{p,q}$

Thus, one can find $\Delta = \partial + \bar{\partial}$ in differentiating the exact form ϵ_2 such that,

1. $\partial\epsilon_2$
2. $\bar{\partial}\epsilon_2$
3. $\partial\bar{\partial} + \bar{\partial}\partial = 0 \begin{cases} \quad where\ Poincaré\ Lemma\ holds\ for\ \partial\ and\ \bar{\partial} \\ for\ \epsilon_2\ in\ \Delta\epsilon_2\ complex\ differential\ it\ is\ \partial\bar{\partial}\ lemma \end{cases}$

Thus, for the $\partial\bar{\partial}\ lemma$, one can satisfy compact manifolds as Kähler provided in the consequence of Hodge Theory, if one corresponds, the $\Delta\epsilon_2$ norm then, for the compact Kähler, a global form of this lemma holds [6-13].

Let L be a compact Riemannian manifold, then for the relation: $\epsilon_1 = \epsilon_2 + \epsilon_3$. When $\Delta\epsilon_3 = 0$ then there exists exactly one $\epsilon_2 - form$ for the De Rham Cohomology class in $H_{dR}^{K}(L)$. Then for the space of the harmonic $(\epsilon_3)\ k - forms\ L$ is isomorphic to $H^k\left(L, \overline{\mathcal{R}}\right)$ taking the sheaf Cohomology for $K^{th} -$ Betti numbers in each of such finite spaces. Thus in this case it can be assumed that the manifold $L \cong S$ for the Abelian group of $\mathcal{R}$ [5-8, 10-16].

Therefore, taking the complex manifold (Kähler) K having a exact form $\Omega^{p,q} \ni \epsilon_2$ the $\partial\bar{\partial}\ lemma$ takes the bidegree (1,1) form of $\Omega^{p,q}\ \forall p,q \geq 1$ for a relation with the De Rham Cohomology such that in the $k - forms$ one can get the exact form of the Kähler $\Omega^k(K)$ whose class is zero is De Rham Cohomology for $H_{dR}^{p+q}(K,\mathbb{C})$ has also the $\partial\bar{\partial} = 0$. This bidegree form is essential for the Kähler potential for $[\epsilon]$ such that in the case of the relation $\epsilon_1 \cong [\epsilon]$, the potential is defined [7-11],

$$\epsilon_1 = i^{-2}\partial\bar{\partial}\rho$$

Where for the Kähler manifold (K, ϵ_1) for the potential of Kähler to be defined as ρ for $[\epsilon]$, there exists the neighborhood μ of $\bar{\rho}$ where $\bar{\rho}$ is a local Kähler potential for the exact differential of $(K,\mathbb{C})$ such that, in compact forms of Kähler one can get the ϵ_2 form of the potential in the local potential $\bar{\rho}$ for the form in $\mu \subset K$ [4-9, 7-13]

$$i\partial\bar{\partial}(K,\mathbb{C}) \equiv \Omega^{1,1}(K)$$

For the $\partial\bar{\partial}$ —manifolds, when it has been assumed the compact space to be L previously, where the compact Kähler denoted as K then its not difficult to say that [12-18],

$$L \approx K$$

Conclusion:

For the bidegree form of the ddbar lemma; the most important consequences can be seen as the Kahler potential for the Kahler manifolds, more specifically that manifold is compact. While taking the harmonic form and sheaf theoretic method into consideration, the related identities can be easily proved where at the end the manifold has been established as Kahler that also in compact form for the De Rham Cohomology.

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