Oligopoly Model and Its Applications in International Trade

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Abstract Each firm in the oligopoly plays off of each other in order to receive the greatest utility, expressed in the largest profits, for their firm. When analyzing the market, decision makers develop sets of strategies to respond the possible actions of competitive firms. In international stage, firms are competitive and they have different business strategies, their interaction becomes essential because the number of competitors is increased. This paper will provide an examination in international trade balance and public policy under Cournot's framework. The model shows how the oligopolistic firm can decide the business strategy to maximize its profit given others' choice, and how the public maker can find out the optimal tariff policy to maximize its social welfare. The discussion in this paper can be significant for both producers in deciding their quantities needed to be sold in not only domestic market but also international stage in order to maximize their profits and governments in deciding the tariff rate on imported goods to maximize their social welfare.

Keywords: Cournot model, international trade, public policy, oligopoly.

1 Introduction

It may be unusual that countries simultaneously import and export same type of goods or services with their international partners (intra-industry trade). However, in general, there are a range of benefits of intra-industry trade offering businesses and countries engaging in it. The benefits of intra-industry trade have been obvious because it reduce the production cost that can be beneficent to consumers. It also gives opportunity for businesses to benefit from the economies of scale, as well as use their comparative advantages and stimulates innovation in industry. Beside to benefits from intra-industry trade, the role of government is also important by using

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its power to protect domestic industry from dumping. Government can apply tariff barrier on imported goods to foreign manufacturers with the aim of increasing the price of imported goods and making them more expensive to consumers. In this international background, managers need to decide the quantity sold in not only domestic market but also other markets under tariff barrier from foreign countries. We consider a game in which the players are firms, nations and strategies are choices of outputs and tariffs. The appropriate game-theoretic model for international trade is the non-cooperate game. The main methods to analyze the strategies of players in this model are developed by the theoretical model: "Cournot Duopoly" - the subject of increased interest in recent years. The target of this paper is to examine the application of Cournot oligopoly analysis to non-collusive firms' behavior in international stage and suggest to decision makers the necessary outcome to maximize their profits as well as the best policy in tariff rate applied by the government. We develop the quantity-setting model under classical Cournot competition in trade theory to find out the equilibrium production between countries in the case that tariffs are imposed by countries to protect its domestic industry and prevent dumping from foreign firms. Section II recalls the Cournot oligopoly model in background. Section III develops the 2-market models with 2 firms competing in the presence of tariff under Cournot behaviors and examines the decision of Governments on tariff rate in considering to its social welfare. In Section III, we can realize the impact of tariff difference on equilibrium price and the quantity of production between 2 countries. Moreover, both governments tend to decide the same tariff rate for importing goods with the aim of maximizing its welfare benefits. Section IV analyzes the model, in general, with n monopolist firms competing in the international trade stage. When n become larger, the difference between equilibrium prices will be equal to the difference between tariff rates as country which imposes the higher tariff rate will have the higher equilibrium price in its domestic market. In addition to that, there will be no difference between the total quantities each firm should produce to maximize its profits when the number of trading countries (or firms) becomes larger. Section IV also considers to welfare benefits of countries and the decision of governments on tariff rates to maximize its domestic welfare. In this section, we also find out that if there is any agreement between countries to reduce its tariff on imported goods, the social welfare in all country could be higher. Section V contains concluding remarks.

2 Review of Cournot Oligopoly Model

Cournot Oligopoly Model is a simultaneous-move quantity-setting strategic game of imperfect quantity competition in which firms (main players), assumed to be perfect substitutes with identical cost functions compete with homogeneous products by choosing its outputs strategically in the set of possible outputs with any nonnegative amount, and the market determines the price at which it is sold. In Cournot oligopoly model, firms recognize that they should account for the output decisions of their rivals, yet when making their own decision, they view their rivals' output as fixed. Each firm views itself as a monopolist on the residual demand curve – the demand left over after subtracting the output of its rivals. The payoff of each firm is its profit and their utility functions are increasing with their profits. Denote cost to firm i of producing q_i units: $C_i(q_i)$, where $C_i(q_i)$ is convex, nonnegative and increasing, given the overall produced amount $(Q = \sum_i q_i)$, the price of the product is $p(Q)$ and $p(Q)$ is non-increasing with Q. Each firm chooses its own output q_i, taking the output of all its rivals q_{-i} as given, to maximize its profits: $\pi_i = p(Q)q_i - C_i(q_i)$.

The output vector $(q_1, q_2, ..., q_n)$ is a Cournot Nash Equilibrium if and only if (given q_{-i}):

$\pi_i(q_i, q_{-i}) \geq \pi_i(q_i', q_{-i})$ for all i.

The first order condition (FOC) for firm i is given by:

$$\frac{\partial \pi_i}{\partial q_i} = p'(Q)q_i + p(Q) - C_i'(q_i).$$

To maximize the firm's profit, the FOC should be 0:

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Leftrightarrow p'(Q)q_i + p(Q) - C_i'(q_i) = 0$$

The Cournot-Nash equilibrium is found by simultaneously solving the first order conditions for all n firms.

Cournot's work to economic theory "ranges from the formulation of the concept of demand function to the analysis of price determination in different market structures, from monopoly to perfect competition"(Xavier Vives,1989). The Cournot model of oligopolistic interaction among firms produces logical results, with prices and quantities that are between monopolistic (i.e. low output, high price) and competitive (high output, low price) levels. It has been successful to help understanding international trade under more realistic assumptions and recognized as the cornerstone for the analysis of firms' strategic behaviour. It also yields a stable Nash equilibrium, which is defined as an outcome from which neither player would like to change his/her decision unilaterally.

3 The basic 2-markets model under tariff

3.1 Trade balance under tariff of the basic 2-factors model

This section will develop a model in which 2 export-oriented monopolist firms in 2 countries. One firm in each country (no entry) produces one homogeneous good. In the home market, $Q_d \equiv x_d + y_d$, where x_d denotes the home firm's quantity sold in the home market and y_d denotes the foreign firm's quantity sold in the home market. Similarly, in the foreign market, $Q_f \equiv x_f + y_f$, where x_f denotes home firm's quantity sold abroad and y_f denotes foreign firm's quantity in its market. Domestic demand $p_d(Q_d)$ and foreign demand $p_f(Q_f)$ imply segmented markets. Firms choose quantities for each market, given quantities chosen by the other firm. The main idea is that each firm regards each country as a separate market and therefore chooses the profit-maximizing quantity for each country separately. In the detection of dumping, each government applied a tariff fee in exporting goods from one country to the other, let t_d be the tariff imposed by Home government to Foreign firm and t_f be the tariff imposed by Foreign government to Home firm to prevent this kind of action and protect its domestic industry (mutual retaliation). Home and Foreign firms' profits can be written as the surplus remaining after total costs and tariff cost are deducted from its total revenue:

$\pi_d = x_d p_d(Q_d) + x_f p_f(Q_f) - C_d(x_d, x_f) - t_f x_f$
$\pi_f = y_d p_d(Q_d) + y_f p_f(Q_f) - C_f(y_d, y_f) - t_d y_d$

We assume that firms in 2 countries exhibit a Cournot-Nash type behavior in 2 markets. Each firm maximizes its profit with respect to own output, which yields the zero first-order conditions and negative second-order conditions. To simplify, we suppose that the demand function is linear with quantity sold in both markets and the slope of both function is -1. Home firm and Foreign firm have fixed costs f and f_1, respectively, and total costs of each firm are quadratic functions with quantities produced:

$p_d(Q_d) = a - (x_d + y_d)$
$p_f(Q_f) = a - (x_f + y_f)$
$C_d(x_d, x_f) = f + \frac{1}{2}k(x_d + x_f)^2$
$C_f(y_d, y_f) = f_1 + \frac{1}{2}k(y_d + y_f)^2$

Where: $a > 0$ is the total demand in the Home market as well as in the Foreign market when the price is zero. Assume that a can be large enough to satisfy the positive value of price and optimal outputs of firms.

$k > 0$ is the slope of the marginal cost function with quantity produced.

From the above equation system, we can reach the first-order and second-order conditions:

$$\begin{cases} \frac{d\pi_d}{dx_d} = a - (2x_d + y_d) - k(x_d + x_f) & = 0 \\ \frac{d\pi_d}{dx_f} = a - (2x_f + y_f) - k(x_d + x_f) - t_f & = 0 \\ \frac{d\pi_f}{dy_d} = a - (x_d + 2y_d) - k(y_d + y_f) - t_d & = 0 \\ \frac{d\pi_f}{dy_f} = a - (x_f + 2y_f) - k(y_d + y_f) & = 0 \\ \frac{d^2\pi_d}{d^2x_d} = \frac{d^2\pi_d}{d^2x_f} = \frac{d^2\pi_f}{d^2y_d} = \frac{d^2\pi_f}{d^2y_f} = -(k+2) < 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_d + x_f = \frac{2a - t_f}{2k+2} - \frac{y_d + y_f}{2k+2} \\ y_d + y_f = \frac{2a - t_d}{2k+2} - \frac{x_d + x_f}{2k+2} \end{cases} \qquad (1)$$

Because the second-order conditions of π_d with respect to x_d, x_f and π_f with respect to y_d, y_f are both negative, then equation (1) shows the reaction functions (best-response functions) for both firms. For any given output level chosen by foreign firm $(y_d + y_f)$ and given tariff rate t_f, the best-response function shows the profit-maximizing output level for home firm $(x_d + x_f)$ and vice versa.

Next, we will derive the Nash equilibrium in this model $(x_d^*, y_d^*, x_f^*, y_f^*)$ by solving the above equation system:

$$\begin{bmatrix} 0 & k & 1 & k+2 \\ k & 0 & k+2 & 1 \\ 1 & k+2 & 0 & k \\ k+2 & 1 & k & 0 \end{bmatrix} \begin{bmatrix} x_d \\ y_d \\ x_f \\ y_f \end{bmatrix} = \begin{bmatrix} a \\ a-t_f \\ a-t_d \\ a \end{bmatrix} \text{ or } A.u = b.$$

We can use the Crammer's rule to solve for the elements of u by replacing the i-th column of A by vector b to form the matrix A_i; then $u_i = |A_i|/|A|$. We have:

$$x_d^* = \frac{\begin{vmatrix} a & k & 1 & k+2 \\ a-t_f & 0 & k+2 & 1 \\ a-t_d & k+2 & 0 & k \\ a & 1 & k & 0 \end{vmatrix}}{|A|} = \frac{a}{2k+3} + t_d \frac{2k^2+4k+3}{3(2k+1)(2k+3)} + t_f \frac{k(4k+5)}{3(2k+1)(2k+3)}$$

$$y_d^* = \frac{\begin{vmatrix} 0 & a & 1 & k+2 \\ k & a-t_f & k+2 & 1 \\ 1 & a-t_d & 0 & k \\ k+2 & a & k & 0 \end{vmatrix}}{|A|} = \frac{a}{2k+3} - t_d \frac{(4k+3)(k+2)}{3(2k+1)(2k+3)} - t_f \frac{2k(k+2)}{3(2k+1)(2k+3)}$$

$$x_f^* = \frac{\begin{vmatrix} 0 & k & a & k+2 \\ k & 0 & a-t_f & 1 \\ 1 & k+2 & a-t_d & k \\ k+2 & 1 & a & 0 \end{vmatrix}}{|A|} = \frac{a}{2k+3} - t_d \frac{2k(k+2)}{3(2k+1)(2k+3)} - t_f \frac{(4k+3)(k+2)}{3(2k+1)(2k+3)}$$

$$y_f^* = \frac{\begin{vmatrix} 0 & k & 1 & a \\ k & 0 & k+2 & a-t_f \\ 1 & k+2 & 0 & a-t_d \\ k+2 & 1 & k & a \end{vmatrix}}{|A|} = \frac{a}{2k+3} + t_d \frac{k(4k+5)}{3(2k+1)(2k+3)} + t_f \frac{2k^2+4k+3}{3(2k+1)(2k+3)}$$

At this point, Home firm is producing an output of x_d^* in Home' market and x_f^* in Foreign's market, Foreign firm is producing an output of y_d^* in Home's market and y_f^*in Foreign's market. If Home firm produces x_d^* in Home' market and x_f^* in Foreign's market, then the best response for foreign firm is to produce y_d^* in Home' market and y_f^* in Foreign's market. Therefore, $(x_d^*, y_d^*, x_f^*, y_f^*)$ is the best response of firms to each other and neither firm has an incentive to derive its choice or the market will be in equilibrium. The equilibrium price in each market will be:

$$p_d^*(Q_d) = a - (x_d^* + y_d^*) = a\frac{2k+1}{2k+3} + t_d \frac{k+3}{3(2k+3)} - t_f \frac{k}{3(2k+3)} \tag{2}$$

$$p_f^*(Q_f) = a - (x_f^* + y_f^*) = a\frac{2k+1}{2k+3} - t_d \frac{k}{3(2k+3)} + t_f \frac{k+3}{3(2k+3)} \tag{3}$$

Moreover, the first-order-conditions and second-order-conditions of $p_d^*(Q_d)$ and $p_f^*(Q_f)$ with t_d and t_f are:

$$\begin{cases} \dfrac{dp_d^*(Q_d)}{dt_d} = \dfrac{k+3}{3(2k+3)} > 0 \,, \dfrac{d^2 p_d^*(Q_d)}{d^2(t_d)} = -\dfrac{1}{(2k+3)^2} < 0 \\ \dfrac{dp_d^*(Q_d)}{dt_f} = -\dfrac{k}{3(2k+3)} < 0 \,, \dfrac{d^2 p_d^*(Q_d)}{d^2(t_f)} = -\dfrac{1}{(2k+3)^2} < 0 \\ \dfrac{dp_f^*(Q_f)}{dt_d} = -\dfrac{k}{3(2k+3)} < 0 \,, \dfrac{d^2 p_f^*(Q_f)}{d^2(t_d)} = -\dfrac{1}{(2k+3)^2} < 0 \\ \dfrac{dp_f^*(Q_f)}{dt_f} = \dfrac{k+3}{3(2k+3)} > 0 \,, \dfrac{d^2 p_f^*(Q_f)}{d^2(t_f)} = -\dfrac{1}{(2k+3)^2} < 0 \end{cases}$$

Thus, in homogeneous condition of other factors, when the tariff tax imposed by Home country increases (t_d increases), the equilibrium price of this good in Home market increases ($p_d^*(Q_d)$ increases) and the equilibrium price in Foreign market decreases ($p_f^*(Q_f)$ decreases) with declining rates as its negative second–order–conditions of $p_d^*(Q_d)$ and $p_f^*(Q_f)$ with t_d and t_f. Similarly, in homogeneous condition of other factors, when the tariff tax imposed by Foreign country increases (t_f increases), the equilibrium price of this good in Home market decreases ($p_d^*(Q_d)$ decreases) and the equilibrium price in Foreign market increases ($p_f^*(Q_f)$ increases) with declining rates as its negative second –order –conditions $p_d^*(Q_d)$ and $p_f^*(Q_f)$ with t_d and t_f. These results can be explained by the fact that, increased tariff will affect to the price of imported goods equivalently and because of this, domestic industry benefits from a reduction of threat of competition, they are not forced to improve their productivity and reduce their prices, this will lead to higher price facing to domestic consumers. Besides that, from (2) and (3) we can verify the effects of the difference in tariff rate on prices:

$$p_d^*(Q_d) - p_f^*(Q_f) = \frac{1}{3}(t_d - t_f) \Leftrightarrow \frac{p_d^*(Q_d) - p_f^*(Q_f)}{t_d - t_f} = \frac{1}{3}.$$

Thus, the proportion between the difference in equilibrium prices of 2 countries and the difference between tariff rates is $\frac{1}{3}$ and if the tariff rate of a country is higher than the other, the equilibrium price in this country will be higher with homogeneous other conditions.

Finally, the equilibrium total productions of 2 firms, Z_d^* and Z_f^*, are:

In Home firm:

$$Z_d^* \equiv x_d^* + x_f^* = \frac{2a}{2k+3} + \frac{t_d}{(2k+1)(2k+3)} - \frac{2t_f(k+1)}{(2k+1)(2k+3)}$$

In Foreign firm:

$$Z_f^* \equiv y_d^* + y_f^* = \frac{2a}{2k+3} - \frac{2t_d(k+1)}{(2k+1)(2k+3)} + \frac{t_f}{(2k+1)(2k+3)}$$

So, $Z_d^* - Z_f^* = \dfrac{t_d - t_f}{2k+1}$.

That means the difference in tariff rates will affect to the difference between optimal quantities of productions in 2 firms in homogeneous condition of other factors. Firm in country which has the higher tariff rate will produce the larger output than the other within country which has the lower tariff rate although it has to face with higher equilibrium price in its market. With the same elasticity of demand in 2 markets, export – oriented firm which has to face the lower tariff from imported country will have more incentive to manufacture the larger amount of its goods due to the lower input cost for each unit of output produced.

3.2 Welfare policy within 2 countries

Brander and Krugman (1983) examined welfare effects under free entry to consumers only. However, the government considered welfare to not only its citizens but domestic industry and its revenue. So, the government imposes a proper tariff on imported goods to prevent dumping from other countries with the aim of maximizing its welfare. The optimal tariff rate imposed to import goods is determined to maximize the welfare of one country (W). The total welfare of a country can be determined as the sum of consumer surplus, tax revenue and firm profit:

W = Consumer Surplus (CS) + Tax Revenue (TR) + Firm Profit (π)

Consumer Surplus (CS) = $\int_0^Q p(Q)dQ - p(Q)Q = \frac{1}{2}Q^2$.

Tax Revenue (TR) = Tariff rate × Imported good amount.

Firm Profit (π) = Total Revenue – Total Cost – Tariff Cost.

In Home country: $W_d = CS_d + TR_d + \pi_d = \frac{1}{2}\left(Q_d^*\right)^2 + t_d y_d^* + \pi_d^*$

In Foreign country: $W_f = CS_f + TR_f + \pi_f = \frac{1}{2}\left(Q_f^*\right)^2 + t_f x_f^* + \pi_f^*$

Using the first-order and second-order conditions in $W_d$andW_f to find out the best-response functions of t_d and t_f:

$$\begin{cases} \frac{dW_d}{dt_d} = 0 \\ \frac{dW_f}{dt_f} = 0 \end{cases} \Leftrightarrow \begin{matrix} Q_d^* \frac{dQ_d^*}{dt_d} + y_d^* + t_d \frac{dy_d^*}{dt_d} + \frac{d\pi_d^*}{dt_d} = 0 \\ Q_f^* \frac{dQ_f^*}{dt_f} + x_f^* + t_f \frac{dx_f^*}{dt_f} + \frac{d\pi_f^*}{dt_f} = 0 \end{matrix} \Leftrightarrow \begin{cases} t_d = -\frac{Q_d^* dQ_d^*/dt_d + y_d^* + d\pi_d^*/dt_d}{dy_d^*/dt_d} \\ t_f = -\frac{Q_f^* dQ_f^*/dt_f + x_f^* + d\pi_f^*/dt_f}{dx_f^*/dt_f} \end{cases}$$

We can express the reaction functions for both governments in deciding their tariff rate as $t_d = f(t_f)$and $t_f = f(t_d)$. When the foreign country increases the tariff rate, the home country needs to decrease the tariff rate in order to maximize its welfare and vice versa.

Proposition 1: In Nash equilibrium, both countries will impose the same tariff rates to imported goods: $t_d^* = t_f^* = t^*$ (Proof: See Appendix A).

However, whether this Nash equilibrium is Pareto efficiency or not, we need to examine the case that 2 nations collude to reduce its tariff barrier on imported goods.

When governments collude to impose the same tariff rate on imported goods: $t_d^* = t_f^* = t$, the social welfare in each country is:

In Home country:

$$W_d = \frac{1}{2}Q_d{}^2 + ty_d + \pi_d = \frac{(2a-t)[2a(k+2)+t(k+1)]}{2(2k+3)^2} - f_d.$$

FOCs with t: $\frac{dW_d}{dt} = \frac{-2a-2t(k+1)}{2(2k+3)^2} < 0$ with $t > 0$.

In Foreign country:

$W_f = \frac{1}{2}Q_f{}^2 + tx_f + \pi_f = \frac{(2a-t)[2a(k+2)+t(k+1)]}{2(2k+3)^2} - f_f.$

FOCs with t: $\frac{dW_f}{dt} = \frac{-2a-2t(k+1)}{2(2k+3)^2} < 0$ with $t > 0$.

Thus, W_d and W_f increase when t decreases, or social welfares in both countries can increase if they agree to reduce its tariff rate. That means Pareto efficiency is not satisfies if two nations complete with each other to protect its domestic industry and maximize its social welfare.

In the Nash equilibrium, two countries impose the same tariff rate on the export country and firms do not have an incentive to unilaterally deviate by altering their output levels: the chosen quantity maximizes the profits of each firm given the quantities chosen by the other firm. Two firms decide the same total amount of its product quantity and the difference in firms' profit depends only on the difference in fixed costs. However, because this Nash equilibrium is not Pareto efficiency, if both countries collude to decide the lower tariff rate on imported goods; the social welfare will be increased.

4 Model expansion

4.1 Trade balance under tariff of the n-factors model

In this section, we expand the model with n monopolist firms assuming oligopolistic competition in the international trade stage and reconsider the choice of optimal policy instruments. Let consider that there are n identical countries, and that each country has one firm i producing commodity Z_i. There is tariff fees incurred in exporting goods from one country to the others. Each firm regards each country as a separate market and therefore chooses the profit-maximizing quantity for each country separately. Each firm has a Cournot perception: it assumes the other firm will hold output fixed in each country. Firm i $(i = 1,2,3,\ldots,n)$ produces output x_{ii} for domestic consumption and output x_{ij} in country j. Firm i has fixed costs f_i and total costs (C_i) of each firm are quadratic functions with quantities produced. The tariff rate imposed by country i in imported goods is t_i where $t_i \geq 0$. Using $p_i(Q_i)$ to denote the demand function in country i and $Q_i = \sum x_{ji}$ be the total quantity sold in market i. Then, profits of firm i can be written, respectively, as:

$$\pi_i = \sum_{j=1}^{n} x_{ij} p_j(Q_j) - C_i(Z_i) - \sum_{j=1,n,j\neq i} t_j x_{ij}$$

Firm i maximize its profit by producing quantity $Z_i = \sum_{j=1}^{n} x_{ij}$ such that it satisfies the FOCs:

$$\frac{d\pi_i}{dx_{ij}} = p_j(Q_j) + x_{ij}\frac{dp_j}{dx_{ij}} - \frac{dC_i(Z_i)}{dx_{ij}} - t_j = 0 \ (j = \overline{1,n}, j\neq i)$$

$$\frac{d\pi_i}{dx_{ii}} = p_i(Q_j) + x_{ii}\frac{dp_i}{dx_{ii}} - \frac{dC_i(Z_i)}{dx_{ii}} = 0$$

Suppose that: $p_j(Q_j) = a - Q_j \ \forall j \in 1,2,3,\ldots,n$.

$$C_i(Z_i) = f_i + \frac{1}{2}kZ_i{}^2$$

We have n^2 equation and n^2 variables, the reaction function of firm i (Z_i) is a function of $(n-1)$ other firms. By solving n^2 equation, we can find out the equilibrium point of firms. From some calculations (See Appendix B), we have the following results:

The total quantity Z_i each firm i should produce to maximize its profits is:

$$Z_i^* = \sum_{j=1}^{n} x_{ij}^* = \frac{na}{nk+n+1} + \frac{t_i}{nk+1} - (nk+2)\frac{\sum_{j=1}^{n} t_j}{(nk+1)(nk+n+1} \ (i = 1,2,\ldots,n)$$

The optimal quantity Q_i^* sold in country i is:

$$Q_i^* = \sum_{j=1}^{n} x_{ji}^* = \frac{na}{nk+n+1} - \frac{n-1}{n+1}t_i + \frac{k(n-1)}{(n+1)(nk+n+1)}\sum_{j=1}^{n} t_j$$

The equilibrium price in country i is:

$$p_i^* = a - Q_i^* = \frac{a(nk+1)}{nk+n+1} + \frac{n-1}{n+1}t_i - \frac{k(n-1)}{(n+1)(nk+n+1)}\sum_{j=1}^{n} t_j$$

The optimal quantity each firm i produce in domestic market i is:

$$x_{ii}^* = p_i^* - kZ_i^*$$

The optimal quantity each firm i produce in foreign market j is:

$$x_{ij}^* = p_j^* - kZ_i^* - t_j$$

From these above results, we have the difference in equilibrium prices as well as quantity produced between 2 countries x and y:

$$p_x^* - p_y^* = \frac{n-1}{n+1}(t_x - t_y)$$

When $n \to \infty$, $\frac{n-1}{n+1} \to 1$, the difference between equilibrium prices will be equal to the difference between tariff rates as country which imposes the higher tariff rate will have the higher equilibrium price in its domestic market. Moreover, the difference in total quantity produced by 2 firms, x and y, that maximize its profits:

$$Z_x^* - Z_y^* = \frac{1}{nk+1}(t_x - t_y)$$

Thus, when $n \to \infty$, $Z_x \to Z_y$ means the difference between the total quantities each firm should produce to maximize its profits is insignificant when the number of country (or firm) becomes larger.

4.2 Welfare policy within n countries

The optimal tariff rate imposed to import goods is determined to maximize the welfare in this country (W_i). The total welfare of a country can be determined as the sum of consumer surplus, tax revenue and firm profit:

$$W_i = ConsumerSurplus(CS_i) + TaxRevenue(TR_i) + FirmProfit(\pi_i)$$

Consumer Surplus of country i $(CS_i) = \int_0^{Q_i} p(Q_i)dQ_i - p(Q_i).Q_i = \frac{1}{2}{Q_i}^2$

TaxRevenue $(TR) = t_i \sum_{j=1, j\neq i}^{n} x_{ji} = t_i(Q_i - x_{ii}) = t_i(2Q_i + kZ_i - a)$

FirmProfit $(\pi_i) = \sum_{j=1}^{n} x_{ij}p_j(Q_j) - C_i(Z_i) - \sum_{j=1, j\neq i}^{n} t_j x_{ij}$

Thus, the total welfare of country i will be determined as:

$W_i = \frac{1}{2}Q_i^2 + t_i(2Q_i + kZ_i - a) + \pi_i$

The equilibrium tariff rates in country i $(i = 1,2,3\ldots,n)$ satisfy the FOCs on W_i:

$$\frac{dW_i}{dt_i} = Q_i\frac{dQ_i}{dt_i} + (2Q_i + kZ_i - a) + t_i\left(2\frac{dQ_i}{dt_i} + k\frac{dZ_i}{dt_i}\right) + \frac{d\pi_i}{dt_i} = 0$$

$\Leftrightarrow t_i = -\dfrac{Q_i\frac{dQ_i}{dt_i} + (2Q_i + kZ_i - a) + \frac{d\pi_i}{dt_i}}{2\frac{dQ_i}{dt_i} + k\frac{dZ_i}{dt_i}}$ $(i = 1,2,3,\ldots,n)$ (The best-response functions of t_i).

Proposition 2: In Nash equilibrium, both countries will impose the same tariff rates to imported goods: $t_1 = t_2 = t_3 = \ldots. = t_n = t^*$. (Proof: See Appendix C).

However, whether this Nash equilibrium is Pareto efficiency or not, we need to examine the case that nations collude to reduce its tariff barrier on imported goods.

When governments collude to impose the same tariff rate on imported goods: $t_i = t \; \forall i = 1,n$, the social welfare in country i is:

$$W_i = \frac{1}{2}{Q_i}^2 + t_i(2Q_i + kZ_i - a) + \pi_i$$
$$= \frac{[na-(n-1)t][a(nk+n+2)+(n-1)(k+1)t]}{2(nk+n+1)^2} - f_i$$

FOCs: $\dfrac{dW_i}{dt} = \dfrac{-2(n-1)a - 2t(k+1)(n-1)^2}{2(nk+n+1)^2} < 0$ with $t > 0$.

Thus, W_i increases when t decreases, or social welfares in both countries can increase if they agree to reduce its tariff rate. That means Pareto efficiency is not satisfies if nations complete with each other to protect its domestic industry and maximize its social welfare. In the Nash equilibrium, the tariff rates imposed by countries tend to converge to t^* and no country is willing to change its tariff rate unilaterally because the chosen quantity maximizes the profits of each firm given the quantities chosen by the other firms. Two firms decide the same total amount of its product quantity, that will lead to the unique price among markets and the difference in firms' profit depends only on the difference in fixed costs. However, because this Nash equilibrium tariff rate in a Cournot oligopoly is not Pareto efficient, if some degree of cooperation can be achieved, both countries can be made better off by simultaneous tariff reductions[1]. Although the government budget may be reduced

[1] This result is equivalent to the conclusion in "Game Theory in International Economics", J.McMillan (2008)

by lowering the tariff rate on imported goods, the consumer surplus as consumers can enjoy the lower equilibrium price and producer surplus as producers get more profits will compensate this loss.

5 Conclusion

We can find that firms will decide the optimal quantity produced and sold in each international market to maximize its profits in the presence of tariff rate imposed by imported countries. The difference in tariff rates between countries will be proportional to the difference in strategic equilibrium prices as countries that impose higher tariff rate will have higher equilibrium prices. Moreover, although the difference in tariff rates imposed by countries has a significant influence to the difference in total quantity produced by firms in the case of a small number of countries participating to international trade, the difference between the total quantities each firm should produce to maximize its profits is insignificant when the number of country (or firm) becomes larger. Moreover, governments should impose the same tariff rate with imported goods in equilibrium to maximize its national economic welfare. In that case, firms decide the same total quantity produced for maximizing its profit and the equilibrium prices will be identical in every market. Finally, if there is any agreement between countries to reduce tariff on imported goods, the social welfare in all country could be higher as people can enjoy the lower price and firms can get higher profits that will compensate to the reducing government budget.

Appendix

APPENDIX A: (Proof of Proposition 1)

From the equation system, we have the FOCs of the welfare functions equal to zero:

$$\begin{cases} \dfrac{dW_d}{dt_d} = 0 \\ \dfrac{dW_f}{dt_f} = 0 \end{cases} \Leftrightarrow \begin{cases} Q_d^* \dfrac{dQ_d^*}{dt_d} + y_d^* + t_d \dfrac{dy_d^*}{dt_d} + \dfrac{d\pi_d^*}{dt_d} = 0 \\ Q_f^* \dfrac{dQ_f^*}{dt_d} + x_f^* + t_f \dfrac{dx_f^*}{dt_f} + \dfrac{d\pi_f^*}{dt_f} = 0 \end{cases}$$

From the optimal quantity produced by firms, equilibrium prices and quantity sold in markets, we can derive:

$Q_d^* + \frac{1}{3}t_d = Q_f^* + \frac{1}{3}t_f$

$Z_d^* - \frac{1}{2k+1}t_d = Z_f^* - \frac{1}{2k+1}t_f$ and $p_d^* - \frac{1}{3}t_d = p_f^* - \frac{1}{3}t_f$

Thus, there is a unique solution of this equation system: $t_d^* = t_f^* = t^*$

The second-order conditions of the welfare functions:

$$\frac{d^2W_d}{dt_d^2} = \frac{d^2W_f}{dt_f^2} = \left(\frac{dQ_d}{dt_d}\right)^2 + 2\frac{dy_d}{dt_d} + \frac{d^2\pi_d}{dt_d^2}$$

$$= \left(-\frac{k+3}{3(2k+3)}\right)^2 - \frac{2(4k+3)(k+2)}{3(2k+1)(2k+3)} + \frac{2}{9} - \frac{k}{(2k+1)^2(2k+3)^2} < 0$$

Therefore, in Nash equilibrium, both countries will impose the same tariff rates to imported goods: $t_d^* = t_f^* = t^*$.

APPENDIX B: The first-order conditions to maximize firms' profits show:

$$\begin{cases} \dfrac{d\pi_i}{dx_{ij}} = p_j(Q_j) + x_{ij}\dfrac{dp_j}{dx_{ij}} - \dfrac{dC_i(Z_i)}{dx_{ij}} - t_j = 0 (j = \overline{1,n}, j \neq i) \\ \dfrac{d\pi_i}{dx_{ii}} = p_i(Q_j) + x_{ii}\dfrac{dp_i}{dx_{ii}} - \dfrac{dC_i(Z_i)}{dx_{ii}} = 0 \end{cases}$$

Suppose that: $p_j(Q_j) = a - Q_j \quad \forall j \in 1,2,3,\ldots,n$ and $C_i(Z_i) = f_i + \frac{1}{2}kZ_i^2$

We have n^2 equation and n^2 variables:

$$\begin{cases} \dfrac{d\pi_i}{dx_{ij}} = p_j(Q_j) + x_{ij}\dfrac{dp_j}{dx_{ij}} - \dfrac{dC_i(Z_i)}{dx_{ij}} - t_j = 0 \left(j = \overline{1,n}, j \neq i\right) \\ \dfrac{d\pi_i}{dx_{ii}} = p_j(Q_j) + x_{ii}\dfrac{dp_i}{dx_{ii}} - \dfrac{dC_i(Z_i)}{dx_{ii}} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} a - Q_j - x_{ij} - kZ_i - t_j &= 0 \\ a - Q_i - x_i i - kZ_i &= 0 \end{cases} \tag{4}$$

From (4), we have: $\forall i = \overline{1,n}$

$$Z_i^* = \sum_{j=1}^{n} x_{ij}^* = \frac{na}{nk+n+1} + \frac{t_i}{nk+1} - \frac{(nk+2)}{(nk+1)(nk+n+1)}\sum_{j=1}^{n} t_j$$

$$Q_i^* = \sum_{j=1}^{n} x_{ji}^* = \frac{na}{nk+n+1} - \frac{n-1}{n+1}t_i + \frac{k(n-1)}{(n+1)(nk+n+1)}\sum_{j=1}^{n} t_j$$

The equilibrium price in country i will be:

$p_i^* = a - Q_i^*$
$= \frac{a(nk+1)}{nk+n+1} + \frac{n-1}{n+1} t_i - \frac{k(n-1)}{(n+1)(nk+n+1)} \sum_{j=1}^{n} t_j$

From (4), the optimal quantity each firm i produce in domestic market i will be:
$x_{ii}^* = p_i^* - kZ_i^*$.
The optimal quantity each firm i produce in foreign market j is:
$x_{ij}^* = p_j^* - kZ_i^* - t_j$.

APPENDIX C: (Proof of Proposition 2)

The optimal quantity each firm i produce in domestic market i will be:
$x_{ii}^* = p_i^* - kZ_i^*$
The optimal quantity each firm i produce in foreign market j will be:
$x_{ij}^* = p_j^* - kZ_i^* - t_j$
We have, the total welfare of country i will be determined as:
$W_i = \frac{1}{2} {Q_i}^2 + t_i(2Q_i + kZ_i - a) + \pi_i$
Where:

$$\pi_i = \sum_{j=1}^{n} x_{ij} p_j(Q_j) - C_i(Z_i) - \sum_{j=1, j\neq i}^{n} t_j x_{ij} \tag{5}$$

Using the first-order-conditions of W_i:
$\frac{dW_i}{dt_i} = 0 \Leftrightarrow Q_i \frac{dQ_i}{dt_i} + (2Q_i + kZ_i - a) + t_i(2\frac{dQ_i}{dt_i} + k\frac{dZ_i}{dt_i}) + \frac{d\pi_i}{dt_i} = 0$
From (5):
$\frac{d\pi_i}{dt_i} = \sum_{j=1}^{n} \left(\frac{dx_{ij}}{dt_i} p_j(Q_j) + x_{ij} \frac{dp_j(Q_j)}{dt_i} \right) - \frac{dC_i(Z_i)}{dt_i} - \sum_{j=1, j\neq i}^{n} t_j \frac{dx_{ij}}{dt_i}$
$= 2\sum_{j=1}^{n} \frac{dp_j}{dt_i} x_{ij} - kZ_i \frac{dZ_i}{dt_i}$ (By using $x_{ii} = p_i - kZ_i$ and $x_{ij} = p_j - kZ_i - t_j$)

Thus, the first-order-conditions of W_i $(i = 1, 2, 3, \ldots, n)$ can be expressed as:
$\frac{dW_i}{dt_i} = Q_i \frac{dQ_i}{dt_i} + (2Q_i + kZ_i - a) + t_i \left(2\frac{dQ_i}{dt_i} + k\frac{dZ_i}{dt_i} \right) + \frac{d\pi_i}{dt_i} = 0$
$\Leftrightarrow Q_i \frac{dQ_i}{dt_i} + (2Q_i + kZ_i - a) + t_i \left(2\frac{dQ_i}{dt_i} + k\frac{dZ_i}{dt_i} \right) + 2\sum_{j=1}^{n} \frac{dp_j}{dt_i} x_{ij} - kZ_i \frac{dZ_i}{dt_i} = 0$

From these above conditions, we have the best-response functions of t_i:

$$t_i = -\frac{Q_i \frac{dQ_i}{dt_i} + (2Q_i + kZ_i - a) + 2\sum_{j=1}^{n} \frac{dp_j}{dt_i} x_{ij} - kZ_i \frac{dZ_i}{dt_i}}{2\frac{dQ_i}{dt_i} + k\frac{dZ_i}{dt_i}} \qquad (i = 1, 2, 3, \ldots, n)$$

In order to find out the Cournot-Nash equilibrium, we need to solve the equation system of n variables $(t_1, t_2, t_3, \ldots, t_n)$. From the optimal quantity produced by firms, equilibrium prices and quantity sold in markets in Appendix B, we can derive:
$Q_i + \frac{n-1}{n+1} t_i$, $Z_i - \frac{1}{nk+1} t_i$ and $p_i - \frac{n-1}{n+1} t_i$ are constant with $i = 1, 2, 3, \ldots, n$.
Thus, there is a unique Nash equilibrium:$t_1 = t_2 = t_3 = \ldots. = t_n = t^*$.

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