A note on actuator fault detection for one-sided Lipschitz systems

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Abstract. This paper investigates the actuator fault detection for one-sided Lipschitz systems. The feature of the nonlinear function lies that the Lipschitz constant is unknown and one-sided Lipschitz constant is known. The aims is to design an adaptive observer for this system. Then, based on the observer, an algorithm to detect the fault occurrence in the actuator is proposed. An example is simulated to verify the feasibility of the approach.

Keywords: one-sided Lipschitz, adaptive observer, fault detection.

1 Introduction

Recently, the investigation of one-sided Lipshitz systems (OLSs) has drawn considerable attention in the control community. [1] firstly proposed the definition of one-sided Lipschitz in the nonlinear observer design problem. [2] presented the definition of quadratical inner-boundedness (QIB), and the existence conditions of observers were formulated by the form of linear matrix inequalities (LMIs). In order to obtain less conservative conditions, [3] improved the results of [2] by S-procedure method. After that, many works were dedicated to the observer design for OLSs, such as [4–6]. In [4], a stochastic observer design method for OLSs is proposed. [5] considered the unknown input observers for the continuous and discrete systems, respectively. [6] extended the observer design method to singular OLSs.

Generally speaking, fault detection (FD) and reconstruction has always been a hot topic in the study of safety for practical systems. If the fault appears in the actuator, the harm caused by is enormous. Thus, it is essential to detect the actuator fault online. To name a few, there exist many methods to deal with the FD, among which the method based on observer is very popular. Indeed, FD based on observer of Lipschitz systems have been studied extensively in the past decades [7–10]. When turns to OLSs, few works have been done except for [11]. In [11], Li et.al considered the nonlinear function which conforms to be both one-sided Lipschitz and QIB. As pointed out in [12], the plants considered in the mentioned references [2–6, 11] are in fact a subclass of OLSs.

Motivated by the above analysis, we further consider the FD problem for OLSs, where the nonlinear function only satisfies one-sided Lipschitz constraint but not QIB. The remainder of this paper is organized as follows: Section 2 presents necessary basics, main results including the adaptive observer(AO) and FD algorithm are given in Section 3. Section 4 simulates one example to show the validation of the proposed approach.

2 Problem formulation and preliminaries

Consider the system with actuator fault:

$$\begin{cases} \dot{x} = Ax + Bf(Hx,u) + Dg(x,u) + Tu_f, \\ y = Cx, \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R^m$, and $y \in R^q$ are the state, the control input, and the output respectively. $f(\cdot,\cdot) \in R^r$ and $g(\cdot,\cdot) \in R^p$ are both nonlinear matrix functions. The signal $u_f \in R^s$ stands for the unknown actuator fault vector. $A \in R^{n\times n}$, $B \in R^{n\times r}$, $C \in R^{q\times n}$, $D \in R^{n\times p}$, $H \in R^{r\times n}$, and $T \in R^{n\times s}$ are determined matrices.

Assumption 1. $f(\cdot,\cdot)$ is one-sided Lipschitz with respect to w, i.e.,

$$< f(w,u) - f(\hat{w},u), w - \hat{w} > \leq \alpha ||w - \hat{w}||^2, \tag{2}$$

where α is the one-sided Lipschitz constant.

Assumption 2. $g(\cdot,\cdot)$ is Lipschitz with respect to w, i.e.,

$$||g(w,u) - g(\hat{w},u)|| \leq \sigma ||w - \hat{w}||, \tag{3}$$

where σ is the Lipschitz constant but unknown.

Assumption 3. There exist positive definite matrice $P \in R^{n\times n}$, matrices $L \in R^{n\times q}$, $F \in R^{r\times q}$, $W \in R^{p\times q}$ and constant $\varepsilon > 0$ such that

$$(A - LC)^T P + P(A - LC) + 2\alpha (H - FC)^T (H - FC) + 2\varepsilon I \leq 0, \tag{4}$$

$$PB = (H - FC)^T, \tag{5}$$

$$D^T P = WC. \tag{6}$$

Remark 1. Generally, $f(Hx,u)$ is only one-sided Lipschitz in this paper. For the purpose of design, checking the condition of QIB is not so easy. Thus, if we remove this constraint, it will obviously extend the type of the system.

3 Main result

For the system (1), an AO is constructed as follows

$$\dot{\hat{x}} = A\hat{x} + Bf(H\hat{x} + F(y - C\hat{x}),u) + Dg(\hat{x},u) + L(y - C\hat{x}) + \frac{1}{2}\hat{\xi} DW(y - C\hat{x}), \tag{7}$$

with adaption law

$$\dot{\hat{\xi}} = \tau ||W(y - C\hat{x})||^2, \tag{8}$$

where the constant τ is positive.

From (7) and (1), the error system can be obtained

$$\dot{e} = (A - LC)e + B\Delta f + D\Delta g - \frac{1}{2}\hat{\xi} DWCe + Tu_f, \tag{9}$$

where $e = x - \hat{x}$, $\Delta f = f(Hx,u) - f(H\hat{x} + F(y - C\hat{x}),u)$, $\Delta g = g(x,u) - g(\hat{x},u)$.

Theorem 1. *Suppose that Assumptions 1-3 hold and $u_f = 0$, the error system* (9) *is asymptotically stable with $\lim_{t\to\infty} e(t) = 0$, i.e., the system* (7) *with* (8) *is an AO for the system* (1).

Proof. Let $\tilde{\xi} = \xi - \hat{\xi}$, and then $\dot{\tilde{\xi}} = -\dot{\hat{\xi}} = -\tau||WCe||^2$. Here we employ the Lyapunov function candidate as follows

$$V = e^T Pe + \frac{1}{2}\tau^{-1}\tilde{\xi}^2, \tag{10}$$

Taking the derivative of (10) along (9), one can get

$$\begin{aligned}\dot{V} &= 2e^T P\dot{e} + \tau^{-1}\tilde{\xi}\dot{\tilde{\xi}} \\ &= 2e^T P[(A-LC)e + B\Delta f + D\Delta g - \frac{1}{2}\hat{\xi}DWCe] - \tilde{\xi}||WCe||^2 \\ &= 2e^T P(A-LC)e + 2e^T PB\Delta f + 2e^T PD\Delta g - \hat{\xi}e^T PDWCe - \tilde{\xi}||WCe||^2.\end{aligned} \tag{11}$$

By Assumption 1, together with (4) and (5), we have

$$\begin{aligned}2e^T[P(A-LC)e + PB\Delta f] = e^T[&(A-LC)^T P + P(A-LC) \\ &+ 2\alpha(H-FC)^T(H-FC)]e \leq -2\varepsilon e^T e.\end{aligned} \tag{12}$$

It follows from Assumption 2 and (6) that

$$\begin{aligned}2e^T PD\Delta g &= 2(D^T Pe)^T\Delta g = 2(WCe)^T\Delta g \\ &\leq 2||WCe||\,||\Delta g|| \leq 2\sigma||WCe||\,||e|| \\ &\leq 2||\sigma WCe||\,||e|| \leq \frac{\sigma^2}{\varepsilon}||WCe||^2 + \varepsilon||e||^2,\end{aligned} \tag{13}$$

and

$$\hat{\xi}e^T PDWCe = \hat{\xi}e^T(WC)^T WCe = \hat{\xi}||WCe||^2. \tag{14}$$

Substituting (12)-(14) into (11), we have

$$\dot{V} \leq -\varepsilon e^T e + (\frac{\sigma^2}{\varepsilon} - \hat{\xi} - \tilde{\xi})||WCe||^2. \tag{15}$$

Let $\xi = \dfrac{\sigma^2}{\varepsilon}$, then

$$\dot{V} \leq -\varepsilon e^T e. \tag{16}$$

By integrating two sides of (16), we can get

$$\int_0^t \dot{V}(t)dt \leq -\int_0^t \varepsilon e^T(s)e(s)ds. \tag{17}$$

In view of the fact that $V(t) > 0$ and $V(0) < \infty$, it is deduced from (17) that

$$\int_0^t \varepsilon e^T(s)e(s)ds \leq V(0) < \infty. \tag{18}$$

By Barbalat Lemma [13], it is concluded that $\lim_{t\to\infty} \varepsilon e^T e(t) = 0$, which implies that $\lim_{t\to\infty} e(t) = 0$.

Remark 2. From the above proof, it is not difficult to notice that the estimation of unknown parameter $\hat{\xi}$ or $\hat{\sigma}$ may not converge to its original value ξ or σ. To some extend, the error $\tilde{\xi}$ or $\tilde{\sigma}$ is only Lyapunov stable but not asymptotically stable.

It is indeed that Assumption 3 provides the sufficient conditions, under which we can design the AO for the system. However, they are not the standard LMIs or linear matrix equalities (LMEs) when $\alpha \neq 0$, and we need derive another forms.

Theorem 2. *If there exist positive definite matrix $P \in R^{n\times n}$, matrices $Y \in R^{n\times q}$, $F \in R^{r\times q}$, $W \in R^{p\times q}$ and constant $\varepsilon > 0$ such that*

$$\begin{bmatrix} A^T P + PA - YC - C^T Y^T + 2\varepsilon I & (H - FC)^T \\ (H - FC) & -\frac{I}{2|\alpha|} \end{bmatrix} \leq 0, \tag{19}$$

$$PB = (H - FC)^T, \tag{20}$$

$$D^T P = WC, \tag{21}$$

then (4)-(6) *hold.*

Proof. Let $Y = PL$ and by using the Schur complement, one can prove that (19)-(21) are the sufficient conditions of (4)-(6).

Remark 3. It is obvious that (19) is LMI while (20) and (21) are LMEs. We can use Scilab [14] to solve the feasible solutions of the constraints of (19)-(21).
Remark 4. If the actuator has no fault, the designed AO (7) with (8) is valid. In other words, the state x of the above system (1) can be asymptotically recovered by $\hat{x}$. When the fault u_f appears, the AO (7) with (8) can be used to detect the fault.

Theorem 3. *Let $e_y = y - C\hat{x}$ and $\mu > 0$ be the alarm threshold. If the system* (7) *with* (8) *is an AO for the system* (1)*, then the following algorithm*

$$\begin{cases} \textit{if } ||e_y|| \leq \mu, \textit{ the actuator has no fault}, \\ \textit{if } ||e_y|| > \mu, \textit{ the actuator has a fault}, \end{cases} \tag{22}$$

can be used to detect the fault u_f.

The proof is direct and omitted here due to the length of this paper. It is worth noting that the alarm threshold μ is very essential in the design of practical systems and affects the accuracy of the FD algorithm.

4 Numeric Simulation

Consider the system (1), where

$$A = \begin{bmatrix} 0.90 & 0.21 \\ 0.35 & -1.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.10 \\ 0.72 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 1.20\ 0.81 \end{bmatrix}, \quad D = \begin{bmatrix} 0.21 \\ 0.46 \end{bmatrix},$$

$$f(Hx,u) = 0.5\sin(Hx) - (Hx)^{\frac{1}{3}}, \quad g(x,u) = 2sin(x), \quad \alpha = 0.5.$$

Solving the conditions (19)-(21) yields

$$P = \begin{bmatrix} 833.01 & -3.54 \\ -3.54 & 1.62 \end{bmatrix}, \quad L = \begin{bmatrix} 23.41 \\ 174.02 \end{bmatrix}, \quad F = -79.61, \quad W = 173.12.$$

By Theorem 1, the AO has the following form:

$$\begin{cases} \dot{\hat{x}}_1 = 0.9\hat{x}_1 + 0.21\hat{x}_2 + 0.10(0.5sin(1.2\hat{x}_1 + 0.81\hat{x}_2 - 79.61(y - \hat{x}_1)) - (1.2\hat{x}_1 + 0.81\hat{x}_2 \\ \quad -79.61(y - \hat{x}_1))^{\frac{1}{3}}) + 0.42sin(\hat{x}_1 + 23.41(y - \hat{x}_1) + 18.1776\xi(y - \hat{x}_1) \\ \dot{\hat{x}}_2 = 0.35\hat{x}_1 - 1.2\hat{x}_2 + 0.72(0.5sin(1.2\hat{x}_1 + 0.81\hat{x}_2 - 79.61(y - \hat{x}_1)) - (1.2\hat{x}_1 + 0.81\hat{x}_2 \\ \quad -79.61(y - \hat{x}_1))^{\frac{1}{3}}) + 0.92sin(\hat{x}_1) + 174.02(y - \hat{x}_1) + 39.8176\xi(y - \hat{x}_1) \\ \dot{\hat{\xi}} = 173.12(y - \hat{x}_1) \end{cases}$$

Firstly, we show the simulation result of the AO. The condition $x(0)$ and $\hat{x}(0)$ are chosen as $[0.1\ \ 2]^T$ and $[0.11\ \ 0.2]^T$, and the initial value of $\hat{\xi}$ is 0, the recovered state $\hat{x}$ of the AO is depicted in Fig.1 and Fig.2. The estimation of the unknown parameter ξ is presented in Fig.3. It can be seen that the designed AO is effective.

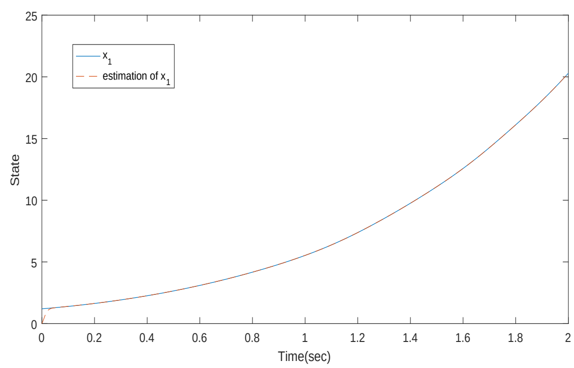

Fig. 1. The state x_1 and the estimation of x_1

When u_f is defined by:

$$u_f = \begin{cases} 50, & 0.4 < t \leq 0.6, \\ 80, & 1 < t \leq 1.2, \\ 0, & \text{else}. \end{cases}$$

The alarm threshold μ is chosen as 0.05. Fig.4 shows that the fault occurs during the time intervals $[0.4,\ 0.6]$ and $[1,\ 1.2]$, which is consistent to the form of function u_f. Thus, the proposed FD algorithm is valid.

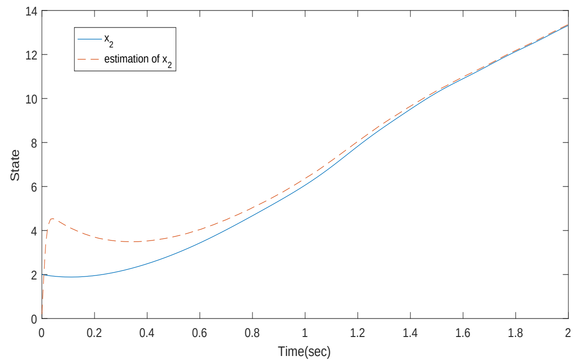

Fig. 2. The state x_2 and the estimation of x_2

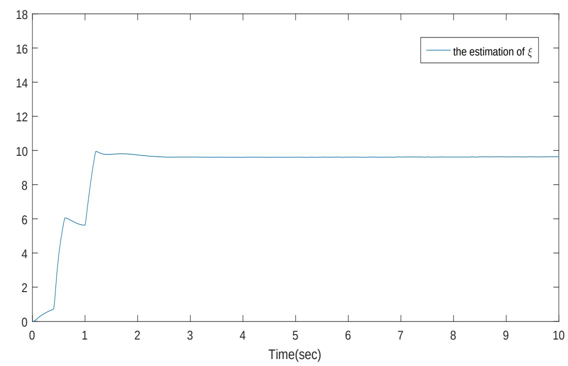

Fig. 3. The estimation of the unknown parameter ξ

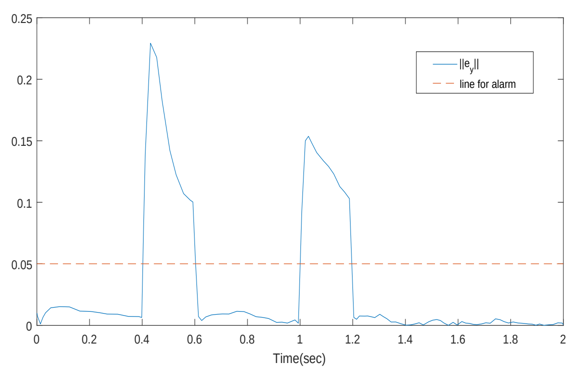

Fig. 4. The time response of $||e_y||$ and the alarm line

5 Conclusion

This paper has studied the FD problem of OLSs. First, we design an AO for the OLS where the nonlinear function is only one-sided Lipschitz. Then, we present FD algorithm based on the designed AO. Finally, an example is given to show that the FD algorithm is effective.

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