Machine Diagnosis Based on Amplitude-Phase Characteristics, Determined from the Experimental Amplitude Spectrum and the Calculated Phase Spectrum

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In the process of diagnosing of machines, due to their universality and efficiency, vibroacoustic methods become more important. The basic signals of these methods are: time courses, amplitude and phase spectra. Time courses and amplitude spectra are often processed and used in current maintenance (diagnostic) practice. Currently, little attention is paid to phase spectra, due to the difficulties encountered in their measurement. However, as is known from the principles of automation, the phase spectrum by Bode's formulas can be calculated based on the amplitude spectrum. Therefore, having the measured amplitude spectrum and the calculated phase spectrum, one can determine the amplitude and phase characteristics of the machine adopted into the automation system. The amplitude-phase characteristics of the "closed" system determined in this way can be converted to the amplitude-phase characteristics of the "open" system, which allows to determine the phase and module reserves of the tested machine. In this situation (always difficult to determine) in the diagnosis process, the diagnostic thresholds result directly from the principles of automation: phase supply and module supply.

Keywords - time courses, phase spectrum, amplitude spectrum, diagnostic, amplitude - phase characteristic.

I. Introduction

Vibroacoustic methods of diagnosis are commonly used in the process of assessing the technical condition of machines and their components. Specialised diagnostic measures for diagnostic tests and specialised algorithms for diagnostic inference are known and used. Standard vibroacoustic signal processing is carried out in the "t" time – the course of the signal is tested, in the "τ" time - the functions of signal correlations are tested, and in the "t*" time - the functions of signal convolutions are tested *($t = t^*$),* as well as signal amplitude spectra are tested in the "ω" frequency domain. It was noticed that this treatment could be a supplement to the analysis of the relationship between the x excitation signals and the y response used in the automation [7]. The object identification frequency methods known in automation, based on spectral transmittance, allowing its frequency characteristics to be determined, are used to a relatively small extent. It is known that frequency characteristics are sensitive to changes in object parameters, and this predisposes them to perform an effective diagnosis of the object [3, 4]. Their very important feature is that they can be experimentally determined with incomplete information about the input signals, as well as the output signals of the object, as well as that the experimental amplitude characteristics can be supplemented by computational phase characteristics. The fact that the amplitude - phase characteristics of the "closed" system can be converted [5] to the amplitude - phase characteristics of the "open" system, which allows to determine the phase and module stability of the stable system, is very important [6]. The automation principles show that the phase reserve is $\Delta\varphi \geq 45^0$ and the module reserve is $\Delta L \geq 6 - 10$ *dB* [6]. System requirements can act as diagnostic thresholds, which will facilitate the diagnosis process because setting the required diagnostic thresholds is always difficult, time consuming and often expensive.

II. Theoretical foundations of the machine description as an automation system

According to the control theory, each machine, unit, element or detail can be reduced to an automation system, on which the $\{x_n(t)\}$ set of signals acts and which is converted into the $\{y_n(t)\}$ set of signals. All the $x_n(t)$ and $y_n(t)$ signals are physical and measurable quantities. Therefore, they fulfill the basic condition sufficient for the existence of equivalent $X_n(j\omega)$ i $Y_n(j\omega)$ Fourier transform [3,7].

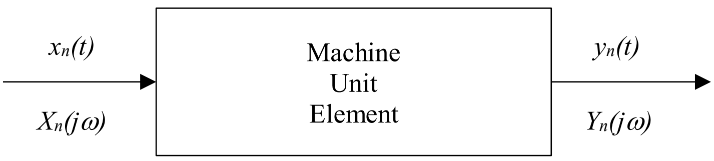


Fig. 1. The machine as an automation system.

The $x_n(t)$ and $y_n(t)$ signals are described in the domain of complex numbers. Therefore:

$$x(t)=A_x(\omega)[cos\omega t+jsin\omega t]=A_x(\omega)e^{j\omega t} \quad (1)$$

$$y(t)=A_y(\omega)\big[cos\big(\omega t+\varphi(\omega)\big)+jsin\big(\omega t+\varphi(\omega)\big)\big]$$

$$=A_y(\omega)e^{j(\omega t+\varphi(\omega))} \quad (2)$$

$A_x(\omega)$, $A_y(\omega)$ – - amplitude of the input and output signal, ω - frequency, t - time, φ - shift phase shift of output function relative to input one.
Finally, each machine, unit or element can be described by means of spectral transmittance [2, 3, 6, 7].

$$G(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{b_m(j\omega)^m+...+b_1(j\omega)+b_0}{a_n(j\omega)^n+...+a_1(j\omega)+a_0}=A(\omega)e^{j\varphi(\omega)} \quad (3)$$

where: $b_0, b_1, ..., b_m$ - transmittance parameters related to its "zeros", $a_0, a_1, ..., a_n$ - transmittance parameters related to its "poles", $A(\omega)$ - amplitude gain $A(\omega)=A_y/A_x$, $\varphi(\omega)$ - phase shift.
In spectral transmittance, the $P(\omega)$ real part and the $Q(\omega)$ imaginary part are distinguished.
Therefore:

$$G(j\omega)=P(\omega)+jQ(\omega) \quad (4)$$

Because:

$$G(j\omega)=A(\omega)e^{j\varphi(\omega)} \quad (5)$$

Then:

$$|G(j\omega)|=A(\omega)=\sqrt{P^2(\omega)+Q^2(\omega)}$$

$$\varphi(j\omega)=arc\ tg\frac{Q(\omega)}{P(\omega)} \quad (6)$$

The relationships between the coordinates of the amplitude - phase characteristics can be determined:

$$Q(\omega)=P(\omega)tg\varphi(\omega)$$
$$P(\omega)=Q(\omega)ctg\varphi(\omega)$$
$$Q(\omega)=A(\omega)sin\varphi(\omega) \quad (7)$$
$$P(\omega)=A(\omega)cos\varphi(\omega)$$

The formulas 4 - 6 and the relationships between coordinates form the basis for determining the frequency characteristics of the machine.
An example of the amplitude - phase characteristics of the machine is shown in Fig. 2.

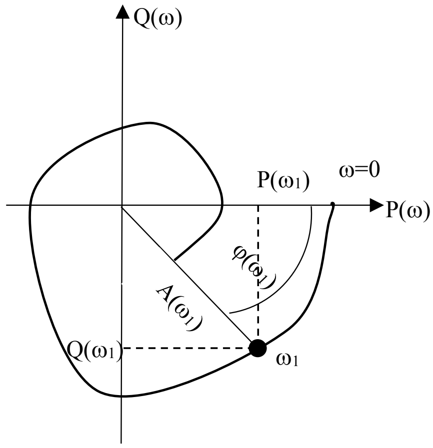


Fig. 2. Amplitude-phase characteristics in the rectangular P and Q coordinates and the polar A and φ coordinates.

where: ω_1 - value input and output signal frequencies, $A(\omega_1)$ - amplitude gain of the output signal relative to the output one, $\varphi(\omega_1)$ - phase shift of the output signal relative to the input one. During the experimental studies, the $A(\omega_n)$ polar coordinates and the $\varphi(\omega_n)$ plotted characteristic are determined, and further according to the $P(\omega_n)$ and $Q(\omega_n)$ needs. The number of the n points should be 17 and more. The fact is that the amplitude-phase characteristics can be determined with incomplete information about the signals, and also when it is possible to experimentally only determine $A(\omega_n)$ without the possibility of experimental determination of $P(\omega_n)$. It is known that the immeasurable $\varphi(\omega_n)$ can be calculated on the basis of Bode's formulas from $A(\omega_n)$ [5].

III. The $\varphi(\omega)$ Calculation Method on the Basis of the $A(\omega)$ Sharped Bands

The $\varphi(\omega)$ phase shift can be calculated on the basis of measured striated spectrum $A(\omega)$ gains according to Bode's relationship [5]:

$$\varphi(\omega)=\frac{1}{\pi}\int_0^{\infty}\frac{lnA(\lambda)}{\lambda-\omega}d\lambda \quad (8)$$

where: $\varphi(\omega)$- phase shift, $A(\lambda)$- amplitude gain of each spectrum band, ω - frequency, λ - integration variable for the next spectrum band.
Each band of the amplitude spectrum can be described by the Gaussian curve equation. The standard Gaussian curve takes the form [1]:

$$A(\lambda)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(\lambda-\omega)^2}{2\sigma^2}} \quad (9)$$

where: σ - standard deviation, ω - positional frequency of a given band.
As the spectrum bands are "thin", it is recommended to use a sharpened Gaussian curve for their description [1]:

$$A(\lambda)=\frac{1}{\sigma^3\sqrt{2\pi}}e^{\frac{-(\lambda-\omega)^2}{2\sigma^4}} \quad (10)$$

For a sharpened Gaussian curve:

$$lnA(\lambda)=ln\frac{1}{\sigma^3\sqrt{2\pi}}-\frac{(\lambda-\omega)^2}{2\sigma^4} \qquad (11)$$

After performing simple calculations and taking into account that [5]:

$$\frac{2\omega}{\pi}ln\frac{1}{\sigma^3\sqrt{2\pi}}\int_0^\infty\frac{1}{\lambda^2-\omega^2}d\lambda=0 \qquad (12)$$

the following result is obtained:

$$\varphi(\omega)=-\frac{\omega}{\pi\sigma^4}\int_0^\infty\frac{\lambda-\omega}{\lambda+\omega}d\lambda \qquad (13)$$

and finally:

$$\varphi(\omega)=-\frac{\omega}{\pi\sigma^4}\int_0^\infty\frac{\lambda}{\lambda+\omega}d\lambda+\frac{\omega^2}{\pi\sigma^4}\int_0^\infty\frac{1}{\lambda+\omega}d\lambda \qquad (14)$$

After integration:

$$\varphi(\omega)=-\frac{\omega}{\pi\sigma^4}(\lambda-\omega|\lambda+\omega|)\Big|_0^\infty+\frac{\omega^2}{\pi\sigma^4}ln|\lambda+\omega|\Big|_0^\infty$$
$$\varphi(\omega)=-\frac{\omega}{\pi\sigma^4}(\lambda-\omega|\lambda+\omega ln(\lambda+\omega)|)\Big|_0^\infty+\frac{\omega^2}{\pi\sigma^4}ln|\lambda+\omega|\Big|_0^\infty \qquad (15)$$

It is assumed that: $\infty\equiv n\sigma$ based on the assumption that $n\rightarrow\infty$. The equation takes the form:

$$\varphi(\omega)=-\frac{\omega}{\pi\sigma^4}(\lambda-\omega ln|\lambda+\omega|)\Big|_0^{n\sigma}+\frac{\omega^2}{\pi\sigma^4}ln|\lambda+\omega|\Big|_0^{n\sigma} \qquad (16)$$

Equation 16 assumes the form:

$$\varphi(\omega)=\left[-\frac{\omega\lambda}{\pi\sigma^4}+2\frac{\omega^2}{\pi\sigma^4}ln|\lambda+\omega|\right]\Big|_0^{n\sigma} \qquad (17)$$

After making simple calculations:

$$\varphi(\omega)=-\frac{n\omega}{\pi\sigma^3}+\frac{2\omega^2}{\pi\sigma^4}ln\left|1+\frac{n\sigma}{\omega}\right| \qquad (18)$$

For sharp bands $n = 3$ [1] is assumed and then the following result is obtained:

$$\varphi(\omega)=\frac{3\omega}{\pi\sigma^3}\ dla\ ln\left|1+\frac{3\sigma}{\omega}\right|\cong\frac{3\sigma}{\omega} \qquad (19)$$

(one segment of the *ln 1+x* expansion in series)

$$\varphi(\omega)=\frac{3\omega}{\pi\sigma^3}-\frac{9}{\pi\sigma^2}\ but\ ln\left|1+\frac{3\sigma}{\omega}\right|\cong\frac{3\sigma}{\omega}-\frac{1}{2}\left(\frac{3\sigma}{\omega}\right)^2 \qquad (20)$$

(two segments of the *ln 1+x* expansion series).
For preliminary calculations, it is finally assumed that:

$$\varphi(\omega)\cong\frac{3\omega}{\pi\sigma^3} \qquad (21)$$

where σ for preliminary calculations, it is finally assumed that.
It is assumed that: σ is equal to the thickness of the *g* band at half of its height [4].

IV. Example of Experimental Studies

The machine (pump) was tested - Fig. 3. The amplitude spectrum was recorded - Fig. 4. The tested bands were described with a sharpened Gaussian curve, and the phase shift was calculated for the tested bands Tab. 1.

TABLE I. PHASE SHIFT OF BANDS

n band	A	g	φ	P	Q
1	1	2,6000	3,1400	0,0000	0,0015
…	…	…	…	…	…
11	0,0011	1,7	0,270	0,0011	0,0003

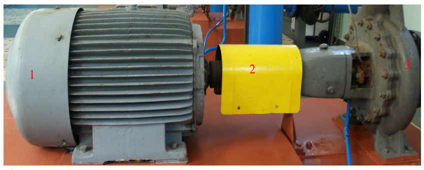


Fig. 3 - View of the tested machine (1 – engine, 2 – coupling, 3 – pump).

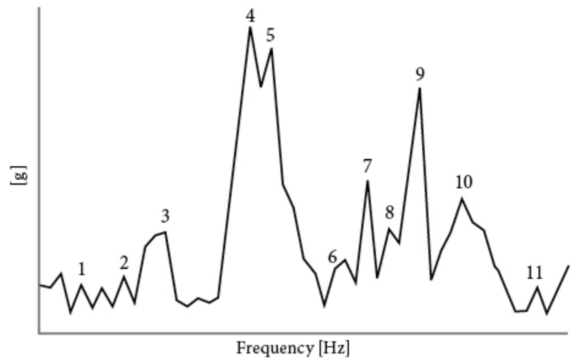


Fig. 4 - Example of measured amplitude spectrum.

Based on the results from Table 1, 11 points of the amplitude - phase characteristics of the machine are determined (Fig. 5).

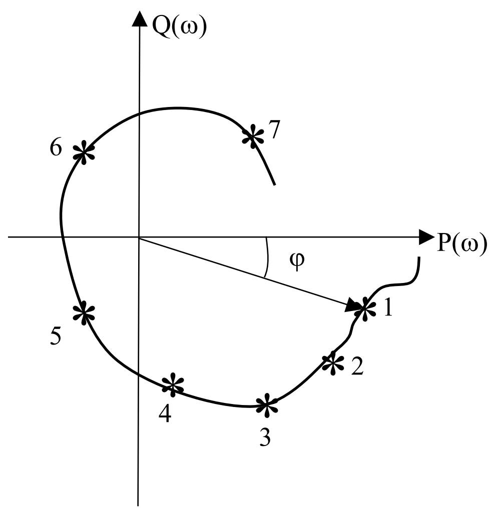


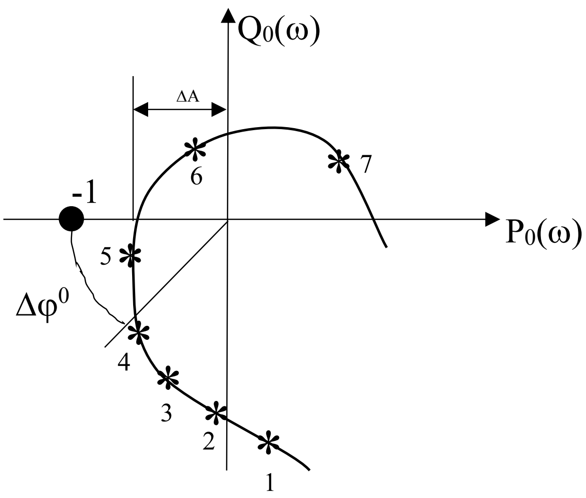
Fig. 5. Amplitude-phase characteristics of the machine adopted into the automation system determined on the basis of the measured amplitude spectrum and the calculated phase spectrum.

The characteristics from Fig. 5 are the characteristics of the "closed" system, which can form the basis for determining the amplitude and phase characteristics of the "open" system. You can designate:

$$P_0=\frac{P(1-P)-Q^2}{(1-P)^2+Q^2}$$

$$Q_0=\frac{Q}{(1-P)^2+Q^2} \tag{22}$$

Therefore, based on the characteristics of Fig. 5, the characteristics of the "open" system are determined Fig. 6.



$$\Delta L=20\log\frac{1}{\Delta A}\,dB$$

Fig. 6. Amplitude-phase characteristics of the "open" system of the machine adopted into the automation system.

The location of the "open" system characteristics relative to the $Q_0=0$, $P_0=-1$ critical point is tested. The $\Delta\varphi$ phase reserve and the ΔL module reserve are determined directly from Fig. 6 for all automation systems including the tested machine, there should be: $\Delta\varphi>45^0$, $\Delta L>6-10\ dB$.

V. Conclusion

Amplitude spectrum is investigated in current operational practice. On its basis, the technical condition of the machine is determined.

The presented method allows to deepen the current algorithms of the technical condition of the machine. The calculated phase spectrum is attached to the measured amplitude spectrum. Then the amplitude and phase characteristics of the "closed" and "open" system are easily determined. The machine is tested as if it was an automation system. The quality indicators of the automation system can be used as diagnostic thresholds. The described method can be of great practical importance, because it allows us to develop existing signal processing with frequency identification used in automation.

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