Dynamic Power Allocation Method Based on ZF Precoding for Multibeam GEO-LEO Co-Existence System

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***Abstract*—Spectrum scarcity has been an obstacle to preventing the development of satellite communication, sharing spectrum between GEO and LEO has proven to be a promising option to resolve it. However, such a system requires GEO as a primary system and LEO as a secondary system that does not cause harmful interference to primary users, and there only consider the inter-satellite interference in the existing literature to maximize the sum rate of the system. This paper proposes a downlink transmission scheme exploiting dynamic power allocation in conjunction with ZF precoding for a multibeam LEO-GEO co-existence satellite system. The motivation behind the proposed design is to jointly optimize the sum rate and user fairness. In our proposed system, we not only consider inter-satellite interference but also consider intra-satellite interference. We formulate a weighted sum rate maximization problem that incorporates SINR requirements for primary users. Since the optimization problem is non-convex, we adopt a closed-form FP approach to transform it. We validate the proposed method with numerical results, it is shown that our scheme can achieve a good balance between system performance and user fairness.**

Keywords—*Dynamic Power Allocation, LEO-GEO co-existence, ZF-Precoding, multibeam satellite, Closed-form FP, User grouping*

I. Introduction

As multimedia broadband service grows by leaps and bounds, an upsurge in the demand for higher data rates intensifies the challenges facing the radio spectrum. There is an active topic of debate about why the available bandwidth is insufficient [1]. And according to the spectrum occupancy measurements, much spectrum appears to be underutilized most of the time. The reason is that inefficient spectrum management, in which each spectrum band is only allocated to one or a few specific services, and within each service, a specific frequency is assigned to a single user in a particular geographic area. Pressed ahead by this fact, cognitive radio (CR) seems to be a promising solution to solve it [2]. CR is a radio that has the availability of acquiring network information and uses it to adapt itself to the surrounding environment. In this way, we can apply it to the satellite system to alleviate the scarcity of the spectrum resource.

There are three paradigms for CR which are *underlay*, *overlay*, and *interweave*, respectively. In the interweave paradigm, secondary users (SUs) only can be served when primary users (PUs) are absent. And in the overlay paradigm, SUs must overhear the transmission of PUs and need to use a part of SUs' power to relay the primary message. Only in the underlay paradigm can SUs and PUs coexist. So in this paper, we adopt the underlay CR. Despite cognitive radio has been extensively studied in terrestrial wireless systems, there still have some new challenges in satellite systems. Cognitive satellite communications allow LEO and GEO to coexist in the same spectrum band which can significantly enhance the efficiency of the spectrum and make full use of available spectrum resources. But the challenging issue here is that GEO is relatively static to the earth, however, the position of LEO changes rapidly over time. Thus, when the LEO is very near to the GEO, especially in the equatorial region, the GEO may receive significant interference from LEO. And to further enhance the system capacity, we consider a multibeam LEO satellite with full frequency reuse which interferes with LEO users impossible to ignore. If these interferences cannot be properly resolved, it will seriously degrade the performance of the satellite system. Thus, the management of interference in the multibeam GEO-LEO coexisting system is of great importance.

To address this issue, [3] exploited the spectrum holes in the space-time dimension and set an 'Exclusion Zone (EZ)' to the LEO satellite, which enforce the LEO beams that fall in the EZ to turn off to protect against harmful interference to GEO satellite ground terminals. And [4] studied an adaptive power control scheme for the coexistence of GEO and LEO satellites. The authors in [5] analyzed the mitigation of multiuser interferences which employ zero-forcing precoding with per antenna power constraints.

Albeit there have been some significant breakthroughs in the study of spectrum sharing between GEO and LEO satellites. There are two aspects of interference management, one is the suppression of interference and the other is the cancellation of interference, and all of these methods aforementioned only focus on one of them.

In this paper, we propose a dynamic power allocation in conjunction with the ZF precoding method for co-existing LEO and GEO systems herein the GEO and LEO satellite are considered as the primary and secondary systems respectively. The novelty of the proposed work lies in the fact that we not only eliminate the interference between LEO users with ZF precoding but also ensure LEO satellites cannot do harmful interferences to GEO users by power allocation. The proposed algorithm can achieve a higher sum rate of the system and guarantee fairness between LEO ground users.

The rest of this paper is organized as follows. In section Ⅱ, we introduce the system model in LEO-GEO co-existing systems. Section Ⅲ presents the algorithm with dynamic power control and ZF precoding techniques. Section Ⅳ provides numerical results to illustrate the superiority of our method. Section Ⅴ gives the conclusion.

II. SYSTEM MODEL

A. *Co-existing satellite system*

In this work, we consider a co-existence multi-beam satellite system in a downlink case where GEO satellite shares spectrum resources with a multi-beam LEO satellite. We assume that GEO satellite serves its users simultaneously within a time slot by using the super-frame technique [6], and LEO users are served via TDMA and SDMA, which is only one user per beam is served at each time slot. An overview of the considered system is depicted in Fig.1.

GEO satellite is relatively static to the earth whereas LEO satellite moves relative to the earth. Thus, the coverage areas of the two satellites will overlap when LEO passes through the coverage area of a GEO beam. Then, there will occur inter-interference which degrade the performance of the satellite system seriously. We suggest using the cognitive radio technique to alleviate interference between LEO and GEO satellites which are in the same frequency band. Additionally, GEO serves as a primary system, and LEO serves as a secondary system.

In the cognitive satellite system, suppose that the gateways of both systems are connected by a high-speed loss-less fiber optic connection to exchange channel state information and satellite ephemeris[4]. As a secondary system, the LEO satellite needs to be coordinated to avoid causing harmful interference to the GEO satellite.

The LEO satellite deploys a multibeam payload with full frequency reuse, and the GEO satellite system is supposed to use a multibeam payload with a conventional seven color reuse pattern.

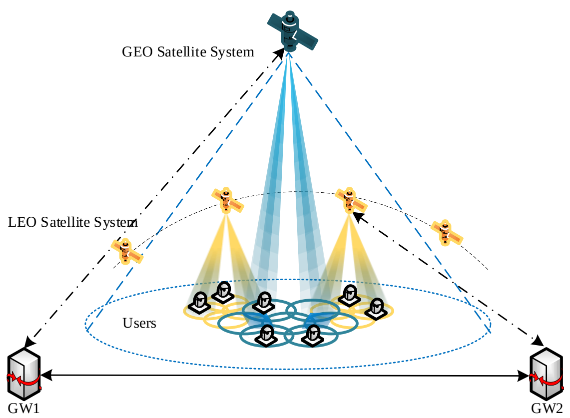

Fig.1 Cognitive satellite system with the GEO and LEO satellite systems.

B. *Signal model*

In this work, we assume that the LEO satellite is equipped with N_{LEO} feeds serving K_{tot} users. The users, each equipped with a single antenna, are uniformly distributed across the coverage area. K_{tot} is assumed to be very large compared to N_{LEO}.

As stated above, following the interference model as shown in Fig.2. The signal received by LEO user k can be written as

$$y_k = \mathbf{h}_k^{L*}\mathbf{x}^L + \mathbf{g}_k^{G*}\mathbf{x}^G + n_k^L, \quad k = 1, \cdots, K_{tot} \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{N_{LEO} \times 1}$ is the transmitted signal, $\mathbf{h}_k^{L*} \in \mathbb{C}^{1 \times N_{LEO}}$ is the channel coefficients between the k-th user and the N antennas of the satellite, $\mathbf{g}_k^{G*} \in \mathbb{C}^{1 \times N_{GEO}}$ is the interference from GEO beams to the LEO user k, and n_k is the zero mean Additive White Gaussian Noise (AWGN) which obeys independent and identically distributed (i.i.d). Without loss of generality, the noise variance will be normalized to one[6]. The first term represents the desired signal for user k, and the second denotes the interference form the GEO satellites.

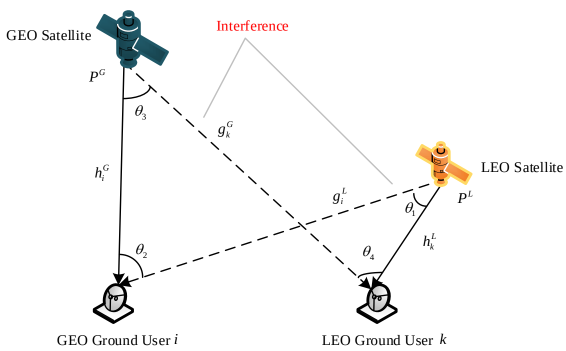

Fig.2 Interference model analysis between LEO and GEO.

As for inter-beam interference, transmit beamforming is an effective technique that separates user data streams into different beamforming directions[7]. Let define s_k, $\mathbf{w}_k \in \mathbb{C}^{N_{LEO} \times 1}$ and p_k^L as, respectively, the unit power data symbol, precoding vector and transmit power scaling factor for user k. Thus, for user k, in detail, the transmitted signal can be read as

$$\mathbf{x}^L = \sum_{k=1}^{K} \sqrt{p_k^L}\,\mathbf{w}_k s_k \tag{2}$$

Thus, the signal received by user k can be rewritten as

$$y_k = \mathbf{h}_k^{L*}\sqrt{p_k^L}\,\mathbf{w}_k s_k + \mathbf{h}_k^{L*}\sum_{j \neq k}\sqrt{p_j^L}\,\mathbf{w}_j s_j + \mathbf{g}_k^{G*}\mathbf{x}^G + n_k \tag{3}$$

where the first term is the useful signal and the second is the interference to user k.

As for the multibeam satellite, the downlink channel matrix $\mathbf{H} \in \mathbb{C}^{K_{tot} \times N}$ is generated as

$$\mathbf{H} = \mathbf{\Phi}\mathbf{B} \tag{4}$$

where $\mathbf{\Phi} \in \mathbb{C}^{K_{tot} \times K_{tot}}$ is the signal phase matrix and $\mathbf{B} \in \mathbb{R}^{K_{tot} \times N_{LEO}}$ represents the multibeam antenna pattern.

Because of the high correlation of signals at the satellite side, there is a common assumption in multibeam channel models is that each user will have the same phase between all transmit antennas due to the long propagation paths[8]. Accordingly, $\mathbf{\Phi}$ is a diagonal square matrix. The diagonal entry is $[\mathbf{\Phi}]_{i,i} = e^{j\phi_i}, \forall i = 1 \ldots K_{tot}$, where ϕ_i is a uniform random variable in $(0, 2\pi]$. The real matrix $\mathbf{B}$ involves antenna gain, path loss, and noise power. Its k,n-th is given by[6]:

$$[\mathbf{B}]_{k,n} = \frac{\sqrt{G_R G_{k,n}}}{4\pi \frac{d_k}{\lambda} \sqrt{\kappa T_R B_W}}, k = 1, \cdots, K_{tot}; n = 1, \cdots, N_{LEO} \quad (5)$$

where d_k is the distance between the k-th user and the satellite, G_R is the gain of receive antenna, and $G_{k,n}$ is the multibeam antenna gain between the k-th user to the n-th antenna on the satellite, λ, B_W, κ, T_R represent the wavelength, bandwidth, Boltzman constant and the clear sky noise temperature at the receiver, respectively.

A common solution for the precoding matrix is the pseudoinverse of the channel matrix [5]. Due to the ZF precoding technique can eliminate the interference completely, after adopting ZF precoding, the signal received by LEO user k can be written as

$$y_k = \mathbf{h}_k^{L*} \sqrt{p_k^L} \mathbf{w}_k s_k + \mathbf{g}_k^{G*} \mathbf{x}^G + n_k \quad (6)$$

So, the SINR at LEO user k can be given by

$$\text{SINR}_k = \frac{p_k^L \left|\mathbf{h}_k^{L*} \mathbf{w}_k\right|^2}{P^G \left|\mathbf{g}_k^{G*}\right|^2 + \sigma^2} \quad (7)$$

then the transmission rate for LEO user k is

$$R_k = B_W \log\left(1 + \text{SINR}_k\right) \quad (8)$$

In (7), apparently, the intra-satellite multiuser interference has been completely eliminated. Whereas, the inter-satellite between LEO and GEO still exists.

It's well known that ZF precoding will achieve optimal performance if the user channels are completely orthogonal to each other[5]. Thus, in the next section, we will propose a new algorithm involving user selection and dynamic power allocation to satisfy the requirement of the orthogonality between users and ensure the service quality of GEO users.

III. Proposed Dynamic Power Allocation with ZF Precoding

ZF precoding technique has a high requirement for the orthogonality of user channels, so selecting the users that are completely orthogonal to each other can exploit the advantage of ZF precoding. But hardly exists a case that users which are served by an LEO satellite happen to be orthogonal to each other. Thus, adopting a proper user selection for ZF precoding is necessitated. Reference [9] proposed a semi-orthogonal user group selection (SUS) algorithm and then used average power allocation, the numerical results have proven that the proposed method can achieve the asymptotic sum rate as the optimal dirty paper coding (DPC) when the number of users approaches infinity. And based on [9], [7] implemented SUS under MIMO broadcast channels and used the water-filling algorithm for power allocation. Extending this result, [5] accounts for the interferences between the two sets, and proposed a new user selection algorithm called Semiorthogonal Interference aware User Allocation algorithm (SIUA).

The aforementioned methods consider user selection, whereas, the power allocation strategies they used don't consider the fairness between users, and what's more, they only apply in the terrestrial network. The algorithm we proposed extends the user selection for ZF precoding to the satellite system and uses the dynamic power allocation that not only can enhance the transmission rate of the LEO-GEO co-existing system but also can ensure the fairness of the service for LEO.

A. *User selection for LEO satellite*

Without loss of generality, we assume that the symmetrical system. That means, we group users into the number of each group is the same as the number of LEO beams. The user selection algorithm details are shown in Algo.1. The procedure works as follows. In Step1), we calculate the orthogonal component of $\mathbf{h}_k$ in the subspace spanned by $\left\{\mathbf{m}_{(1)}, \ldots, \mathbf{m}_{(i-1)}\right\}$. And then in Step2), we select the best user who has the maximum projected norm. In Step4), we dropped off the users who don't satisfy the requirement of orthogonality by using the predefined parameter α. Note, according to [10], we are supposed to choose α wisely because the best value α decrease with the number of users, thus if the amount of users is in the range of 100-100000, the value α should range form $0.2 \sim 0.4$.

B. *Dynamic Power Allocation*

By adopting the user selection algorithm given in the previous section, we can get multiple LEO ground users. And according to the previous discussion, under the premise that maximizing the sum rate of the satellite system is subject to the QoS requirement for the GEO satellite. This means, that for a specific GEO user g, the received SINR should be higher than a predefined threshold

$$\frac{P^G \left|\mathbf{h}_g^{G*}\right|^2}{\sum_{k=1}^{N_{LEO}} g_k^L P_k^L + \sigma^2} \geqslant \gamma_{th} \quad (9)$$

where the interference tolerance threshold γ_{th} we set for each primary user (PU) is -123dBW in the case of the system bandwidth is 500MHz [11].

Based on the interference threshold, the power allocation problem within the LEO user group can be formally stated as the sum rate optimization problem

$$\begin{aligned} &\max_{\mathbf{P}} \sum_{k \in \mathcal{T}} S_k R_k \\ \text{s.t.} \quad &C1: \frac{P^G \left|\mathbf{h}_g^{G*}\right|^2}{\sum_{k=1}^{N_{LEO}} g_k^L P_k^L + \sigma^2} \geqslant \gamma_{th}, \forall g \in K_{GEO} \\ &C2: 0 \leqslant P_i^L \leqslant P_{max}^L, \forall i \in N_{LEO} \\ &C3: 0 \leqslant P^G \leqslant P_{max}^G \end{aligned} \quad (10)$$

where $\mathbf{P} = \left\{P_1^L, P_2^L, \ldots, P_{|\mathcal{T}|}^L\right\}$ represents the proportion of transmit power allocated to LEO ground users in a group, S_k

represents the amount of service requests for user k in group and is used as a weight, K_{GEO} denotes the ground users set of GEO satellite, P_{max}^{L} and P_{max}^{G} is the maximum power of LEO beam and GEO beam respectively, and R_k can be given by(8).

Because there exists a ratio term both in the objective function and the constraint C1, it's can't be solved by using standard convex optimization techniques.

In order to address the issue, we consider the closed-form Fractional programming (FP) approach proposed in [12]. After applying the closed-form FP in the objective function, (9) can be modified as

$$f_r^{\mathrm{CF}}(\mathbf{p},\lambda)=\sum_{i\in\mathcal{T}} S_i\log(1+\lambda_i)-\sum_{i\in\mathcal{T}} S_i\lambda_i+\sum_{i\in\mathcal{T}}\frac{S_i\left(1+\lambda_i\right)\left|h_{i,i}^{L}\right|^2 p_i^L}{P^G\,|\mathbf{g}_i^{G*}|^2+\sigma^2} \tag{11}$$

where λ refers to a set of auxiliary variables $\{\lambda_i\}_{i\in\mathcal{T}}$.

For each LEO ground user, the optimal λ_i^* can be given by

$$\lambda_i^*=\frac{\left|h_{i,i}^{L}\right|^2 p_i^L}{P^G\,|\mathbf{g}_i^{G*}|^2+\sigma^2},\forall i\in\mathcal{T}. \tag{12}$$

And for the constraint C1, as the numerator and the denominator are all positive, so it can be rewritten as

$$P^G\left|\mathbf{h}_g^{G*}\right|^2-\gamma_{th}\times\left(\sum_{k=1}^{N_{LEO}} g_k^L P_k^L+\sigma^2\right)\geqslant 0,\forall g\in K_{GEO} \tag{13}$$

Then, the non-convex optimization can be transformed as follows:

$$\begin{aligned}&\max_{\mathbf{p}}\ f_r^{\mathrm{CF}}(\mathbf{p},\lambda)\\&s.t.(13),C2,C3.\end{aligned} \tag{14}$$

It can be observed that the optimization algorithm is an iterative optimization. Firstly, fix the value of λ and solve the problem(14) iteratively; then update the value of λ according to (12).

IV. SIMULATION RESULTS

In this section, numerical results for the sum rate performance and the fairness performance for the dynamic power allocation in conjunction with ZF precoding in a multibeam LEO-GEO co-existence satellite system are presented. The parameter settings in the simulation are summarized in Table Ⅰ.

In Fig.3, we compare the sum rate performance of different power allocation schemes. It can be observed that the LEO-GEO co-existing scenario does enhance the capacity of the system. And the average power allocation[9] and the water-filling power allocation[10] are worse than our proposed power control method.

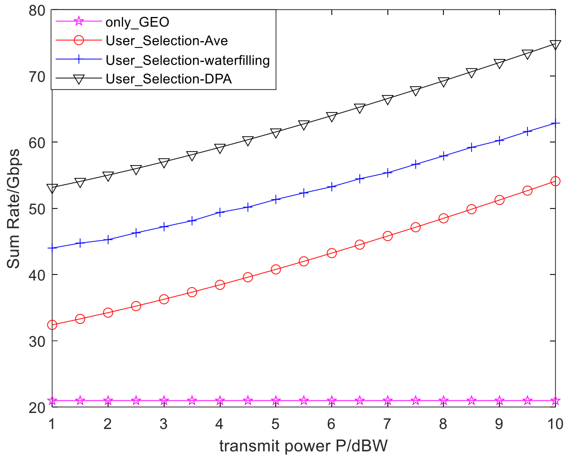

Fig.3 Comparison of sum rate for different schemes

Then, we discuss the fairness of the three power allocation methods. Because the beams of the LEO satellite serve different groups of LEO ground users, the fairness between different user groups can not be ignored. For this, we adopt the Jain's fairness index [13] as follows:

$$\mathrm{FI}=\frac{\left(\sum_{m=1}^{N_g}\sum_{n=1}^{N_{LEO}} R_{m,n}\right)^2}{N_g N_{LEO}\sum_{m=1}^{N_g}\sum_{n=1}^{N_{LEO}}\left(R_{m,n}\right)^2} \tag{15}$$

where R_i denotes the data rate received by the user in the i-th group. Simulation results are given in Fig.4. It can be observed that our proposed method can provide better fairness than others. Because in our proposed method, we used the amount of service requests as a weight in the optimization problem(10), the group with larger service requests will have a higher priority in power allocation which can ensure fairness between users.

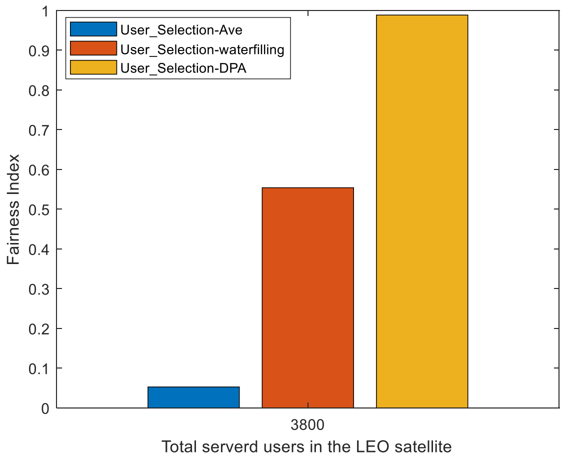

Fig.4 Fairness Index of the three power allocation methods

TABLE I. SIMULATION PARAMETERS

Parameters	*Notations*	*Value*
Bandwidth	B_W	$500MHz$
Frequency band	f	$20GHz\,(Ka)$
Height of GEO satellite	H_{GEO}	$35786km$

Parameters	*Notations*	*Value*
Height of LEO satellite	H_{LEO}	1000km
Antenna diameter of GEO satellite	D_{GEO}	0.6m
Antenna diameter of LEO satellite	D_{LEO}	0.1m
Antenna diameter of receiver user	D_{user}	0.3m
Frequency reuse factor of GEO	K	7
Number of LEO beam	N_{LEO}	19
Number of LEO system users	K_{tot}	3800
The number of groups for LEO users	N_g	200
Number of GEO beam	N_{GEO}	7
Maximum power of LEO beam	P^L_{max}	10W
Maximum power of GEO beam	P^G_{max}	300W
Noise temperature of receiver antenna	T_R	290K
Antenna efficiency	η	55%
The Boltzman constant	κ	$1.38\times10^{-23} J/K$
Gain of receive antenna	G_R	40dB

Algorithm 1: LEO User Selection Algorithm

Input: The set of LEO ground users $\mathcal{T}=\{1,2,\cdots,K_{tot}\}$, the number of LEO beams N, the matrix of channel gain $\mathbf{H}=\begin{bmatrix}\mathbf{h}_1 & \cdots & \mathbf{h}_{K_{tot}}\end{bmatrix}$

Output: The set of selected users $\mathcal{S}$

Initialize $i=1$, $\mathcal{S}=\varnothing$, $N_g=\frac{K_{tot}}{N}$

for group $=1:N_g$

Step1:

for k **in** $\mathcal{T}$ do

if i=1

$\mathbf{m}_k=\mathbf{h}_k$

else

$$\mathbf{m}_k=\mathbf{h}_k-\sum_{j=1}^{i-1}\frac{\mathbf{h}_k\mathbf{m}^*_{(j)}}{\left\|\mathbf{m}_{(j)}\right\|^2}\mathbf{m}_{(j)}=\mathbf{h}_k\left(\mathbf{I}-\sum_{j=1}^{i-1}\frac{\mathbf{m}^*_{(j)}\mathbf{m}_{(j)}}{\left\|\mathbf{m}_{(j)}\right\|^2}\right)$$

end

end

Step2:

(a) selected user index= $\arg\max_{k\in\mathcal{T}}\left\|\mathbf{m}_k\right\|$

(b) **Update** $\mathcal{S}=\mathcal{S}\cup$ {selected user index}

(c) **Update** $\mathbf{m}_{(i)}=\mathbf{m}_{\text{selected usre index}}$

Step3:

Update

$$\mathcal{T}=\left\{k\in\mathcal{T},k\neq\text{selected usre index}\,\middle|\,\frac{\left|\mathbf{h}_k\mathbf{m}^*_{(i)}\right|}{\left\|\mathbf{h}_k\right\|\left\|\mathbf{m}_{(i)}\right\|}<\alpha\right\}$$

$i=i+1$

If $|\mathcal{S}|<N$

Go back to *Step1*

end

V. Conclusion

In this paper, we propose a dynamic power allocation combined with the ZF precoding method for co-existing LEO and GEO systems, in which GEO is considered as the primary system and LEO is considered as the secondary system. Specifically, we formulated this power allocation problem as an optimization problem that maximizes the weighted sum rate for the LEO users in each group subjected to the QoS requirement for the GEO satellite. By using the closed-form Fractional programming technique, the non-convex problem can be transformed into a convex problem. Simulation results show that our proposed method can not only achieve a higher transmission rate than the system which only has a GEO satellite but also can ensure the fairness of the service for LEO users.

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