The Problem of Choosing Criteria in Diversification Models in the Era of the Digital Economy

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Проблема вибору критеріїв в моделях диверсифікації в епоху цифрової економіки

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Abstract

The subject of the research of the article is multi-criteria models of a diversified portfolio that minimize the risks that arise in the era of the digital economy when managing retail chains. The aim of the work is to analyze the problem of choosing criteria in the corresponding multicriteria or vector diversification problems. The article examines the advantages of introducing an additional criterion of entropy maximization into the criteria of the classical two-criteria model of portfolio theory, which characterizes the degree of diversity of the portfolio composition. A complex combination of methods of classical portfolio theory and multicriteria optimization is applied. The results include a comparison of three methods for solving the following problems: criteria convolution, successive concessions, and computer simulation of the Pareto set. Conclusions: the results obtained will be useful for automating the risk management of retail chains.

Keywords 1

Multicriteria problem, optimal portfolio problem, convolution of criteria, method of successive concessions, Pareto set, entropy

1. Introduction

The transition to a digital economy, digital trade in the world in recent years has had a significant impact on the Ukrainian economy as a whole. The massive digital transformation has been accelerated by the COVID-19 pandemic, which has impacted consumer behavior and changed the way business operates. The digitalization of the economy reduces the cost of doing business by automating the relevant processes, but any transformation creates new risks and economic instability. Economic instability leads to a drop in the standard of living and, as a result, negatively affects the activities of trade enterprises, especially in the context of the restoration of Ukraine [1]. Small and medium businesses are especially sensitive to any changes. The decrease in demand for most everyday goods has a painful effect on the activities of small and medium-sized businesses and leads to the emergence of new risks. These risks have a significant impact on reducing the profitability of enterprises. Therefore, it is important for each enterprise to diversify the activities of the enterprise, which includes the expansion of the product range, the reorientation of sales markets and the optimal distribution of goods between divisions of one enterprise.

The traditional portfolio theory of Markowitz [2, 3], in terms of the profitability of the enterprise, proposes to consider the expected profitability of the enterprise as a weighted sum of the expected profitability of the network units, and risk as a deviation from the expected profitability of the enterprise. In the article [2], Markowitz breaks the portfolio formation into 2 stages.

The first stage is the analysis of historical data, which determines the future return R and risk for each division. The second stage is the final formation of the portfolio, taking into account the best

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returns and lower risk. After that, the portfolio is optimized according to the principle: maximizing the return $R \to max$ with an acceptable risk $D > Dopt$ and minimizing the risk $D \to min$ with a given return $R = const$. The solution to the problem is the vector of particles of subdivisions in the distribution of resources. In both cases, the two-criteria problem is reduced to single-criterion problems under certain restrictions.

The basic concept in solving problems of making managerial decisions is the set of alternatives, which, on the one hand, must be sufficiently diverse so that the decision maker (DPR) does not lose the opportunity to choose an alternative that will be in a certain sense the best from her point of view [4]. On the other hand, it must be a set that the decision maker can process. In particular, in [4,5,6,7,8] the basic definitions, various approaches and methods of multicriteria optimization are systematically presented. Let us give some main definitions.

The mathematical formulation of the problem of multicriteria optimization determines the set of alternatives (SA) and the method of its representation (enumeration, ratio, deterministic generation mechanism), the admissible set X (AS) and the vector objective function (VOF)

$$F = (F_1(x), F_2(x), \dots, F_N(x)), \tag{1}$$

which is defined on an admissible set X. One of the options for formalizing the set of alternatives is the Pareto set.

The Pareto set $\tilde{X}$ consists of non-dominated solutions $\tilde{x}$, for each of which there is no feasible solution $x^* \in X$ satisfying the inequalities

$$F_i(x^*) \le F_i(\tilde{x}), \tag{2}$$

where $i = 1,2, \dots N$, among which at least one inequality is strict.

It is often useful to distinguish between two types of multiobjective problem statements, namely an individual problem and a mass problem [8]. An individual problem has fixed parameters of the vector objective function $F = (F_1, F_2, \dots, F_N)$, and a constraint system. In the statement of the mass problem, which has a common name, some parameters are not fixed and are given by notation. For example, the mass problem is the classical two-criteria Markowitz portfolio problem with a vector objective function $F = (R, D)$.

Methods for solving multicriteria (vector) problems are based on different approaches. One of the approaches is the construction of a generalized criterion that aggregates the vector of VOF criteria (1). For example, the method of linear or multiplicative convolution of criteria, the majority criterion, a geometric criterion based on immersion in a metric space. Another approach is to define the lexicographic order of the criteria. Thus, attempts are made to move from a multi-objective task to a single-objective task or a sequence of single-objective tasks with certain restrictions. The choice of the solution strategy affects the resulting solution, as the previous constraints on the solution of the problem change and new constraints are added. Not all methods can guarantee a valid solution. In particular, for certain problems, the linear convolution method does not allow one to obtain a Pareto set. In this regard, the problem of solving multicriteria problems with the help of linear convolution of criteria (LCC) is considered separately [4].

Consider this algorithm. Algorithms of linear convolution of criteria are based on the fact that, with a positive definite VOF, the element $x \in X$ that maximizes (minimizes) linear convolution of criteria

$$F^{\lambda}(x) = \sum_{\upsilon=1}^{N} \lambda_{\upsilon} F_{\upsilon}(x), \tag{3}$$

is pareto-optimal. Here is the vector $\lambda \in \Lambda_N$, where $\Lambda_N = \{\lambda = (\lambda_1, \dots, \lambda_N): \quad \sum_{\upsilon=1}^{N} \lambda_{\upsilon} = 1, \quad \lambda_{\upsilon} > 0, \quad \upsilon = 1,2,\dots,N\}$.

Consider some individual problem with N maximizable criteria and defined on the set of feasible solutions $X = \{x\}$. Let us denote the set of alternatives of this problem by $X^*, X^* \subseteq X$. If for each element $x^* \in X^*$ there is a vector $\lambda^* \in \Lambda_N$ that satisfies the equality $F^{\lambda^*}(x^*) = \max_{x \in X} F^{\lambda^*}(x)$, then it is said that the problem of finding SA X* is solvable using the linear convolution algorithm. If the solvability defined in this way is typical for all individual problems of the mass problem, for each of them it is possible to find the SA using convolution algorithms. This problem is unsolvable with the convolution algorithms, if for the problem under consideration there is an individual problem with SA X* containing such an element $x^* \in X^*$, on which the convolution extremum for $F^{\lambda}(x)$ $\forall \lambda \in \Lambda_N$ will not be reached, i.e. for any $\lambda \in \Lambda_N$, the inequality $F^{\lambda}(x^*) < \max_{x \in X} F^{\lambda^*}(x)$ strictly holds.

Another method for solving problems of multicriteria optimization is the method of successive concessions, which requires preliminary ranking of criteria by significance. At each step k, a single-objective problem with an objective function of rank k is solved. Also, new restrictions are added to the system of restrictions, providing a deviation of the value of criteria from 1 to (k-1) rank by the amount of allowable concession $\delta_l > 0,\ l = 1,2, \dots k-1$ . . The solution of the problem is obtained when N one-objective conditional optimization problems with criteria $F_i(x)$, where $i = 1,2, \dots N$. The final result is the optimal value of the least important criterion, subject to the guaranteed values of the previous criteria. In [9], the effectiveness of applying the method of successive concessions for solving multicriteria problems of diversification of a centralized pharmacy network of different sizes was analyzed, and the stability regions in the space of parameters of the concession method were determined.

Solving problems of multicriteria optimization is a non-trivial task, which is due to the conceptual uncertainty of incomparable vectors. The final decision is always made by the decision maker. To substantiate this choice, it is necessary to evaluate the properties of the obtained solutions when applying different approaches.

Therefore, the purpose of this work is to analyze the problem of choosing a set of criteria and the effectiveness of solving the multi-criteria problem of diversifying a trading network using various methods: successive concessions, linear convolution, and computer simulation.

2. Problem statement and results

Let us give a more detailed mathematical statement of the problem of diversification of the trade network and recall the main definitions. The mathematical statement uses the apparatus of describing multidimensional random variables. The profitability of the network is estimated as

$$R = \sum_{i=1}^{n} r_i x_i,$$

where vector $x^* = \begin{pmatrix} x_1^* \\ \dots \\ x_n^* \end{pmatrix}$ - the share of trade network divisions in the portfolio of network assets, r_i - profitability of the network division, $i = \overline{1,n}$.

Risk D estimated using a variance matrix $W = |\omega_{ij}|$, $\omega_{ij} = cov(x_i, x_j)$ – covariance, $i, j = \overline{1,n}$:

$$D = \sum_{i=1}^{n} \omega_{ii} x_i^2 + 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j,$$

with restrictions

$$0 \le x_i \le 1,\ \sum_{i=1}^{n} x_i = 1.$$

As already mentioned, the classic optimal portfolio model is a two-criteria problem with a vector objective function F=(R,D).

Let us apply the classical model to formally describe the problem of optimal distribution of goods in a wholesale distribution network between branches. It is necessary to determine the share of goods for each branch to ensure maximum profit R for the entire network with minimum risk D. To achieve the best ratio of expected return and risk, it is useful to carry out diversification measures, the effectiveness of which requires research. A feature of the problem is the presence of mutual influence between the network subdivisions.

With this formulation, it is necessary to define the concept of profit and risk, in order to determine the influencing factors and quality criteria for evaluating possible alternatives.

Consider a trade network that has n - trade points (branches). We denote:

ϑ_i - expected value of the sold product in sales prices of the i-th branch for the year (average value of sales for each branch for m years), $i = \overline{1, n}$.

ϑ_{si} - the expected cost of product sales at the purchase prices of the i-th branch for a year (average over m years), $i = \overline{1, n}$.

ϑ_{zi}- the expected amount of expenses of the i-th branch per year (average for m - years), i $= \overline{1, \text{n}}$.

Then $\vartheta_{0i} = \vartheta_{si} + \vartheta_{zi}$ is the cost of the sold goods.

The profitability of the i-th branch will look like this

$$r_i = \frac{\vartheta_i + \vartheta_{0i}}{\vartheta_{0i}}. \tag{4}$$

Let us denote the share of the distributed resource of the i-th branch by

$$x_i = \frac{\vartheta_{0i}}{\sum_{i=1}^{n} \vartheta_{0i}} \tag{5}$$

Then the profitability of the entire enterprise will look like this:

$$R = \sum_{i=1}^{n} r_i x_i. \tag{6}$$

Indeed, $R = \sum_{i=1}^{n} \frac{\vartheta_i - \vartheta_{0i}}{\vartheta_{0i}} * \frac{\vartheta_{0i}}{\sum_{i=1}^{n} \vartheta_{0i}} = \frac{\sum_{i=1}^{n} \vartheta_i - \sum_{i=1}^{n} \vartheta_{0i}}{\sum_{i=1}^{n} \vartheta_{0i}} = \frac{\vartheta \ \ - \vartheta_0}{\vartheta_0}$, where ϑ - is the expected cost of goods sold during the year throughout the enterprise; ϑ_0 - is the expected cost of goods sold during the year throughout the enterprise.

Risks in the formation of an assortment portfolio are taken into account using dispersion:

$$D = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j = \sum_{i=1}^{n} \omega_{ii} x_i^2 + 2 \sum_{i=1}^{n} \sum_{i=1}^{n} \omega_{ij} x_i x_j, \tag{7}$$

where

$\omega_{ij} = cov(x_i, x_j)$ - covariance, $\omega_{ij} = \omega_{ji}$, $i, j = \overline{1, n}$.

The solution is the vector $x^* = \begin{pmatrix} x_1^* \\ \dots \\ x_n^* \end{pmatrix}$. Knowing x_i^*, $i = \overline{1, n}$,, it is possible to obtain the amounts of the distributed resource by branches. From formula (5) we have: $x_i^* = \frac{\vartheta_{si} + \vartheta_{zi}}{\sum_{i=1}^{n} \vartheta_{si} + \sum_{i=1}^{n} \vartheta_{zi}}$.

From here, we will get: $\vartheta_{si} = (\sum_{i=1}^{n} \vartheta_{si} + \sum_{i=1}^{n} \vartheta_{zi}) x_i^* - \vartheta_{zi}$, ϑ_{zi} - the average value of costs for each division for a certain period.

The assessment of the level of diversification is carried out by determining the value of entropy according to the method of K. Shannon, which characterizes the degree of diversity of the system. The introduction of entropy as the third criterion will make it possible to influence the level of diversification, as well as the assortment structure of the portfolio: $E = -\sum_{i=1}^{n} x_i \ln x_i$.

Next, five models of the portfolio diversification of the retail network with different VOF(2) composition are presented.

MODEL 1 corresponds to the problem of two-criteria optimization with the vector objective function Φ_1, which contains the risk criterion D and the entropy criterion E.

It is necessary to find such a vector $x^* = \begin{pmatrix} x_1^* \\ \dots \\ x_n^* \end{pmatrix}$ with known $W = |\omega_{ij}|$, $\omega_{ij} = cov(x_i, x_j)$, that

$$\Phi_1 \quad = (D, E), \tag{8}$$

where

$$D = \sum_{i=1}^{n} \omega_{ii} x_i^2 + 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j \to min,$$

$$E = -\sum_{i=1}^{n} x_i \ln x_i \to max.$$

In case of exchanges on the level of profitability r_p, how to get by an expert

$$R = \sum_{i=1}^{n} r_i x_i \geq r_p.$$

And also

$$0 \leq x_i \leq 1, \ \sum_{i=1}^{n} x_i = 1.$$

MODEL 2 - formalizes the three-criteria optimization problem with the vector objective function Φ_2, which contains criteria: network profitability R, risk D and entropy criterion E.

It is necessary to find such a vector $x^* = \begin{pmatrix} x_1^* \\ \dots \\ x_n^* \end{pmatrix}$ with known $W = |\omega_{ij}|$, $\omega_{ij} = cov(x_i, x_j)$, that

$$\Phi_2 = (R, D, E), \tag{9}$$

where

$$R = \sum_{i=1}^{n} r_i x_i \to max$$

$$D = \sum_{i=1}^{n} \omega_{ii} x_i^2 + 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j \to min$$

$$E = -\sum_{i=1}^{n} x_i \ln x_i \to max$$

In case of exchanges on the level of profitability r_p, how to get by an expert

$$R = \sum_{i=1}^{n} r_i x_i \geq r_p.$$

And also

$$0 \leq x_i \leq 1, \ \sum_{i=1}^{n} x_i = 1.$$

MODEL 3 is a single-criterion problem derived from a classical problem with a vector objective function $\Phi_3 = (R, D)$ due to convolution of criteria in the form of R/D.

It is necessary to find such a vector $x^* = \begin{pmatrix} x_1^* \\ \dots \\ x_n^* \end{pmatrix}$ with known $W = |\omega_{ij}|$, $\omega_{ij} = cov(x_i, x_j)$, that

$$\Phi_3 = R/D \to max, \tag{10}$$

where

$$R=\sum_{i=1}^{n} r_i x_i$$

$$D=\sum_{i=1}^{n} \omega_{ii} x_i^2+2 \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j$$

In case of exchanges on the level of profitability r_p, how to get by an expert

$$R=\sum_{i=1}^{n} r_i x_i \geq r_p.$$

And also

$$0 \leq x_i \leq 1,\ \sum_{i=1}^{n} x_i=1.$$

MODEL 4 represents a two-criterion optimization problem with a vector objective function Φ_4,, which includes the convolution criterion from model 3, i.e. Φ_3, and the entropy criterion E.

It is necessary to find such a vector $x^*=\begin{pmatrix} x_1^* \\ \ldots \\ x_n^* \end{pmatrix}$ with known $W=|\omega_{ij}|,\ \omega_{ij}=cov(x_i, x_j)$, that

$$\Phi_4=(\Phi_3, E), \tag{11}$$

where

$$\Phi_3=R/D \rightarrow max,$$

$$R=\sum_{i=1}^{n} r_i x_i$$

$$D=\sum_{i=1}^{n} \omega_{ii} x_i^2+2 \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j$$

$$E=-\sum_{i=1}^{n} x_i \ln x_i \rightarrow max$$

In case of exchanges on the level of profitability r_p, how to get by an expert

$$R=\sum_{i=1}^{n} r_i x_i \geq r_p.$$

And also

$$0 \leq x_i \leq 1,\ \sum_{i=1}^{n} x_i=1.$$

MODEL 5 is a modification of MODEL 2, and formalizes the problem of two-criteria optimization with the vector objective function Φ_5, which contains criteria: network profitability R, risk D.

It is necessary to find such a vector $x^*=\begin{pmatrix} x_1^* \\ \ldots \\ x_n^* \end{pmatrix}$ with known $W=|\omega_{ij}|,\ \omega_{ij}=cov(x_i, x_j)$, that

$$\Phi_5=(R, D), \tag{12}$$

where

$$R = \sum_{i=1}^{n} r_i x_i \rightarrow max$$

$$D = \sum_{i=1}^{n} \omega_{ii} x_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_{ij} x_i x_j \rightarrow min$$

In case of exchanges on the level of profitability r_p, how to get by an expert

$$R = \sum_{i=1}^{n} r_i x_i \geq r_p.$$

And also

$$0 \leq x_i \leq 1,\ \sum_{i=1}^{n} x_i = 1.$$

All five models described are mass tasks. When working with real numerical data, corresponding individual tasks are formed.

3. Experiments

Experiments were carried out for individual tasks of Models 1 - 5 on the basis of data provided by a decision maker in the trading network.

To justify the choice of the final solution, we will perform the solution by methods related to the construction of a generalized criterion and the concession method for a different set of criteria, and also apply different software and analyze the results.

Numerous experiments were carried out on the same data using various software tools: 1) using the composite gradient method in the "Search for a Solution" MS Excel service and 2) using the developed software in the MATLAB package [10].

Let us first consider the application of the solution method using the generalized criterion of linear convolution of criteria. It is necessary to construct an optimization integral criterion with an objective function of the form

$$C \quad = \sum_{i=1}^{N} \alpha_i\, C_i \quad \rightarrow extr, \tag{13}$$

where ...

C_i – normalized values of the vector objective function (2), that is $F \quad = (C_1, C_2, \dots, C_N)$,

$\sum_{i=1}^{n} \alpha_i = 1$ та $0 \leq \alpha_i \leq 1$-$0 \leq \alpha_i \leq 1$ - constant reflecting the degree of importance of each partial criterion C_i

We build an optimization problem based on MODEL 1 with an objective function

$$\Phi'_1 = -\alpha \quad {}^{D}\!/_{D_{max}} + (1-\alpha) \quad {}^{E}\!/_{E_{max}} \rightarrow max, \tag{14}$$

where

$0 \leq \alpha \leq 1$-constant reflecting the degree of importance of each partial criterion

Optimization problem based on Model 2 with an objective function:

$$\Phi'_2 = \alpha_1\, {}^{R}\!/_{R_{max}} - \alpha_2\, {}^{D}\!/_{D_{max}} + \alpha_3\, {}^{E}\!/_{E_{max}} \rightarrow max, \tag{15}$$

where

$0 \leq \alpha_i \leq 1$-constant reflecting the degree of importance of each partial criterion, $\sum_{i=1}^{n} \alpha_i = 1$.
Optimization problem based on Model 5 with an objective function:

$$\Phi'_5 = -\alpha \quad {}^{D}/_{D_{max}} + (1-\alpha) \quad {}^{R}/_{R_{max}} \to max \tag{16}$$

We will solve the problem by linear convolution of criteria (3) using the composite gradient method in the "Search for a solution" MS Excel service.

Let us formulate an individual task of forming an effective investment portfolio of a trading company with 5 branches. Based on the data on the sale and expenses of this enterprise for 5 years (2017-2021), the distribution vectors of the resource ϑ_s and profitability r were compiled:

$\vartheta_s = (88\,228{,}15;\ \ 189\,947;\ 170\,569;\ 141\,857;\ 99\,669),$

$r = (0{,}0050;\ 0{,}0393;\ 0{,}0123;\ 0{,}0085;\ 0{,}0116).$

Covariance coefficients:

$$\omega_{ij} = \begin{pmatrix} 0{,}000663 & 0{,}0003 & 0{,}000091 & -0{,}000214 & -0{,}000152 \\ 0{,}0003 & 0{,}000011 & -0{,}00033 & -0{,}00001 & 0{,}000024 \\ 0{,}000091 & -0{,}00033 & 0{,}000151 & 0{,}000004 & -0{,}000139 \\ -0{,}000214 & -0{,}00001 & 0{,}000004 & 0{,}000043 & 0{,}000028 \\ -0{,}000152 & 0{,}000024 & -0{,}000139 & 0{,}000028 & 0{,}000135 \end{pmatrix}.$$

Standard deviation according to 2017-2021 data

$$\sigma = (0{,}0257; 0{,}003; 0{,}0123; 0{,}0086; 0{,}0116)$$

We find an efficient portfolio by the method of linear convolution with $\alpha_1 = \alpha_2 = 0{,}5$

$$x^* = (0{,}13; 0{,}29; 0{,}25; 0{,}19; 0{,}14),$$
$$R = \sum_{i=1}^{n} r_i x_i = 0{,}018,$$
$$D = \sum_{i=1}^{n} \omega_{ii} x_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_{ij} x_i x_j = 1{,}12 \text{ E-05},$$
$$\text{E} = 0{,}67819.$$

Next, we will consider the method of successive concessions used to solve multicriteria problems with a preliminary ranking of criteria in terms of significance. At each step, a single-objective conditional optimization problem is solved. At the first step, the objective function is the optimization criterion of the first rank. Constraints coincide with the constraints of the original problem. At each next step k, a single-criteria problem with a target function of rank k is solved and new constraints are added to ensure the deviation of the value of criteria from 1 to k-1 rank by the allowable concession $\delta_l > 0,\ l = 1,2, \ldots k-1$.

Let's demonstrate the work of the method of successive concessions on the example of Model 2 with the vector objective function $F_2 = (R, D, E)$, when ranking the criteria: entropy> risk>income, that is, E>D>R. With the chosen ranking, we obtain the following sequence of single-objective conditional optimization problems.

First step:

$$E = -\sum_{i=1}^{n} x_i \ln x_i \to max$$

$$\begin{cases} R = \sum_{i=1}^{n} r_i x_i \geq r_p \\ 0 \leq x_i \leq 1 \end{cases} \tag{17}$$

The value of E* is the optimal value according to the criterion of the first rank.

Second step. The objective function is to minimize the risk. The condition of deviation of the optimal value E* by the allowable concession $\delta_1 > 0$ is added to the constraints of the original problem:

$$D = \sum_{i=1}^{n} \omega_{ii} x_i^2 + 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j \to min$$

$$\begin{cases} \left|- \sum_{i=1}^{n} x_i \ln x_i - E^*\right| \leq \delta_1 \\ R = \sum_{i=1}^{n} r_i x_i \geq r_p \\ 0 \leq x_i \leq 1, \sum_{i=1}^{n} x_i = 1 \end{cases} \tag{18}$$

The risk value D* is the optimal value according to the criterion of the second rank.

Third step. The objective function is the maximization of income R. The condition of deviation of the optimal value D* is added to the restrictions of the second step problem by no more than the allowable concessions $\delta_2 > 0$

$$R = \sum_{i=1}^{n} r_i x_i \to max$$

$$\begin{cases} \left|- \sum_{i=1}^{n} x_i \ln x_i - E^*\right| \leq \delta_1 \\ \left|\sum_{i=1}^{n} \omega_{ii} x_i^2 + 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j - D^*\right| \leq \delta_2 \\ R = \sum_{i=1}^{n} r_i x_i \geq r_p \\ 0 \leq x_i \leq 1, \sum_{i=1}^{n} x_i = 1 \end{cases} \tag{19}$$

Eight experiments were carried out, the structure of which is presented in Table 1

Table 1

The structure of the experiments

№exp	Model	Content of criteria	Solution method
1	Model,1 Φ'1	E, D	LCC (14)
2	Model 2, Φ'2	E, D, R	LCC (15)
3	Model 5, Φ'5	D, R	LCC (16)
4	Model 1, Φ1	E> D	concession (8)
5	Model 2, Φ2	E> D > R	concession (9)
6	Model 2, Φ2	E> R>D	concession (9)
7	Model 3, Φ3	R, D	multiplicative convolution (10)
8	Model 4, Φ4	E> Φ3	concession (11)

The results of Experiment 1 on model 1 with the solution by the method of linear convolution of criteria and computer simulation are shown in Fig. 1. The optimal solution was obtained as a vector X=(0.124; 0.201; 0.032; 0.445; 0.197), on which the optimal criteria values are achieved: Min D=2.0467e-05, Max R=0.015, Max E=1.3729.

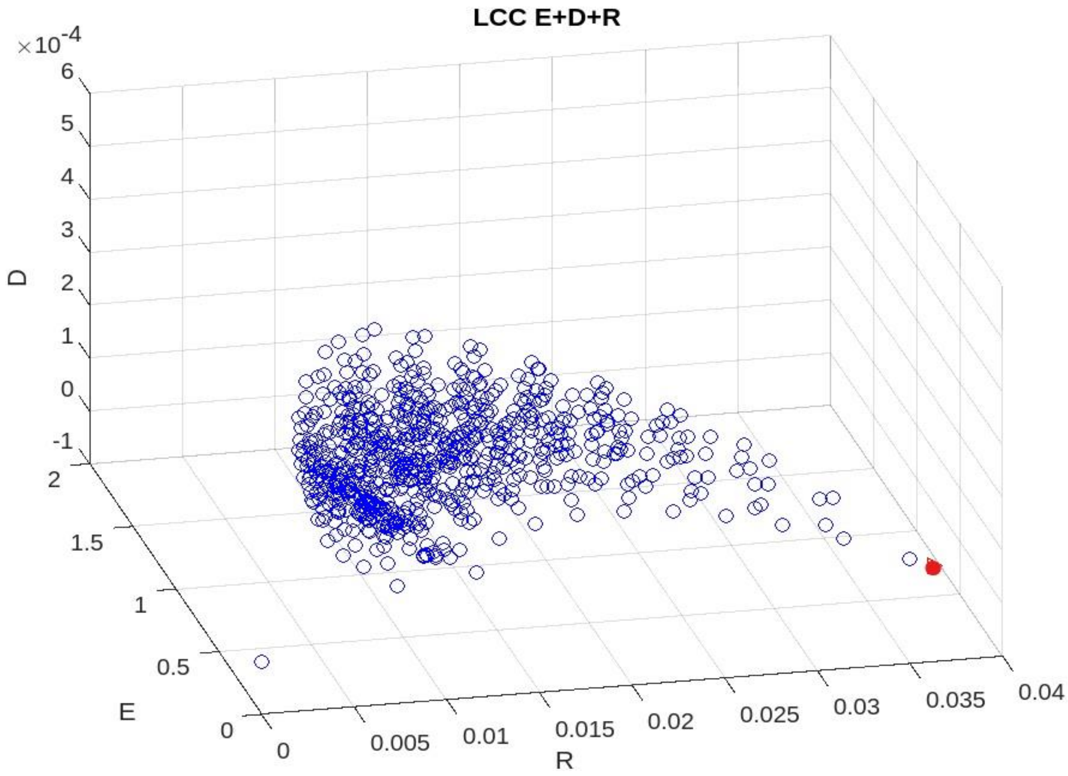

Figure 1: Experiment 1: Model 1(LCC) F1(6) Entropy + Risk

The second experiment consisted in solving the problem according to model 2 by the method of linear convolution of criteria. The results of computer simulation are shown in Fig.2. The optimal solution was obtained as a vector X=(0.001; 0.939; 0.001; 0.035; 0.023), where the optimal criteria values are achieved: Min D=0.000013, Max R=0.037512, Max E=0.2772.

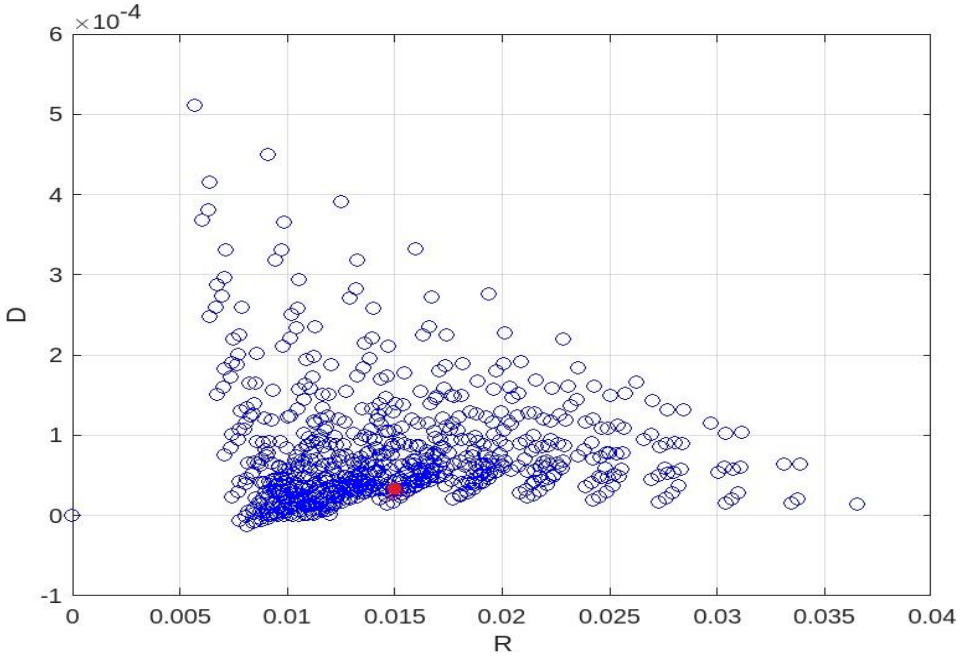

Figure 2: Experiment 2: Model 2(LCC) F'2(7) Entropy+Risk+Return - Projected onto the Plane (Income, Risk) (R, D)

The results of Experiment 5 turned out to be extraordinary - the solution by the concession method according to Model 2 led to an unacceptable solution X=(0.095; 0.277; 0.069; 0.325; 0.233), on which the criteria values are achieved: Min R=0.000030, Max E=1.469858. The results of computer simulation for this case are presented in Fig.3.

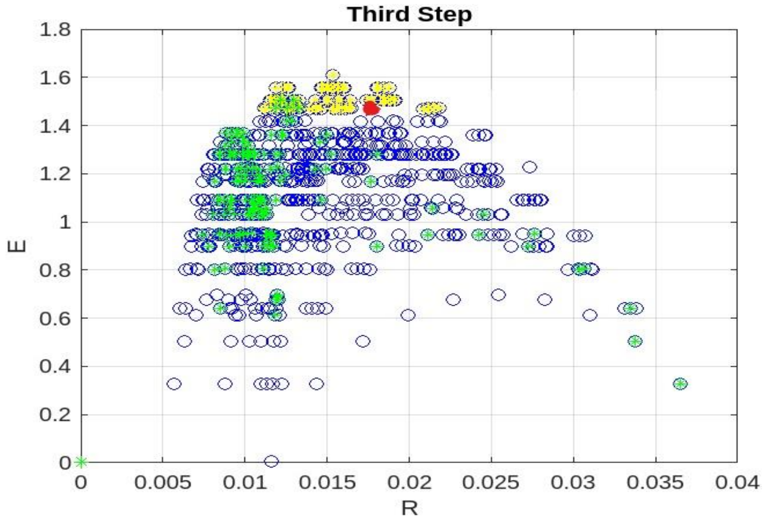

Figure 3: Experiment 5: Model 2 (concessions method): E> D> R. Yellow color - concessions zone by entropy value E, green color - concessions zone by risk value D.

The results of experiment 7 refer to Model 3 and are presented in Fig.4. The optimal solution has the form X=(0.038; 0.206; 0.063; 0.679; 0.013), on which the optimal criteria values are achieved: Min D=0.000028, Max R=0.015, Max E=0.9425.

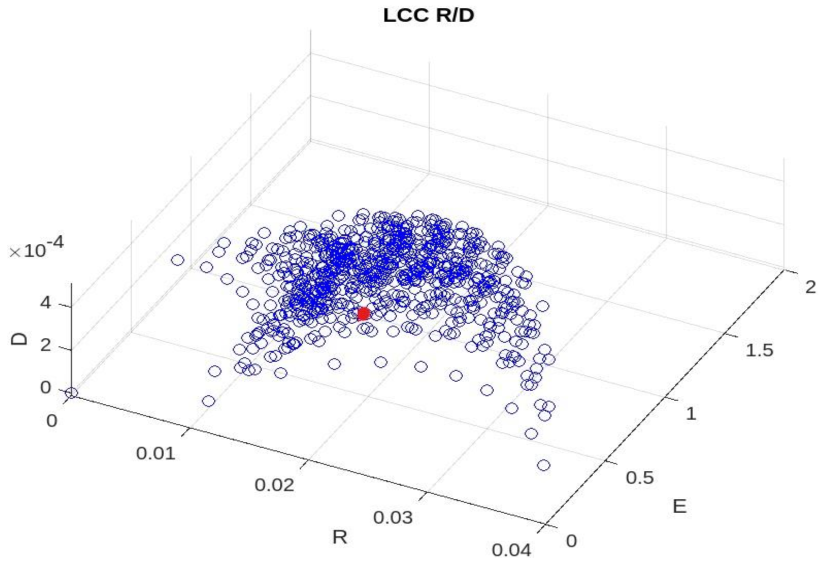

Figure 4: Experiment 7, Model 3, with R/D Convolution

The results of experiments that were carried out on the same data using different software tools are shown in Table 2.

Table 2
Results of solving the problem of enterprise diversification

MS Excel				MATLAB			
№exp	Max E	Max R	Min D	№exp	Max E	Max R	Min D
1	0,69088	0,015	4,09E-05	1	1,3729	0,015	2,05 E-05
4	0,62907	0,015	2,39E-05	4	1,456153	0,015122	2,50 E-05
8	0,62907	0,015	2,39E-05	8	0,973765	0,015	1,40 E-05
5	0,62907	0,01575	2,63E-05	**5**	**1,469858**	**0,017697**	**3,40 E-05**
6	0,62907	0,02067	4,45E-05	6	1,452399	0,017933	3,30 E-05
2	0,61663	0,02301	5,98E-05	2	0,27729	0,037512	1,30 E-05
3	4,05E-08	0,0393	1,10E-05	3	0,000011	0,0393	1,45 E-05
7	**3,80E-08**	**0,0393**	**1,10E-05**	**7**	**0,942531**	**0,015**	**2,80 E-05**

Analysis of Table 2 proves that the results obtained are related to the Pareto set in all experiments and are non-dominated and incomparable, with the exception of the non-Pareto result of experiment 7. Comparison of results 7 and 8 proves that the risk is reduced by introducing the entropy criterion. That experiment 8 dominates result 7 proves the importance of including entropy. Comparison of the results of experiments 1,2 proves the need to introduce a profitability criterion. Analysis of the results obtained when applying the method of linear convolution of criteria proves that only the use of all three criteria allows you to get an adequate result (Experiments 1,2,3,7). Experiments 4 and 8 showed tolerance for entropy and risk results when applying the concession method, and when using LCC, the result gives an improvement in entropy and worsening in risk (Experiments 1 and 4, 8). The result of experiment 5 - in the concession method, we set a 10% possibility of deviation. The numerical method did not allow us to find this result. The deviation in the second step is 40%. That is, by analogy with the concept of insolvability by the convolution method, this example can be considered as insolvability by the concession method.

The discrepancy between the solutions in MS Excel and MATLAB indicates the features of the numerical solution and the problem of choosing software. Built-in MS Excel services and built-in MATLAB functions apply numerical methods that produce standard features of the numerical solution, such as the accumulation of errors of numerical methods and calculation errors.

4. Discussion and Conclusion

To formalize the problem, five models are proposed that differ in vector objective functions, both in the quantity and quality of the selected criteria.

Two directions for solving multicriteria problems are considered. The first direction is the construction of a generalized criterion based on the component of the objective function vector of a multicriteria problem. The second direction is a step-by-step solution, considering the lexicographic order of criteria in terms of importance for the decision maker. The results of the experiments led to general conclusions: the use of the entropy criterion can reduce the risk; MODEL 2 makes it possible to obtain the highest profitability when solving both by the method of linear convolution of criteria and by the method of concessions; multiplicative convolution produces a non-Pareto solution.

Thus, using the conclusions of the classical portfolio theory, proven by experience and time, in this paper we have developed a methodology for the efficient distribution of resources between the branches of the trading network, which takes into account the expected profitability and diversification of distribution and minimizes risks.

The scientific novelty of this work is the formalization based on the portfolio theory and methods of multi-criteria optimization of wholesale distribution network diversification models.

The practical value is that the obtained results of real data for the network have demonstrated the possibility of using the developed tool for automatic allocation of resources in the form of pareto-optimal portfolios in order to minimize risks. Among the directions for further research is a series of experiments with different ways of formalizing risk in portfolio models and searching for appropriate analytical dependencies.

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6. References

[1] The Digital Economy: A Look Into the 2022 Digital Frontier, May 11, 2022. URL:https://www.uschamber.com/on-demand/technology/digital-economy-the-global-competition-to-write-the-rules
[2] H. M. Markowitz, K. Blay, Risk–Return Analysis: The Theory and Practice of Rational Investing (a four-volume series), McGraw-Hill, 2014.
[3] J. B. Guerard (ed.), Handbook of Portfolio Construction. Contemporary Applications of Markowitz Techniques, Springer, New York, Dordrecht, Heidelberg, London, 2010, p.808. URL: http://doi: 10.1007/978-0-387-77439-8.
[4] V. A. Perepelitsa, I. V. Kozin, E. V. Tereshchenko, Zadachi classifikatsii i formirovanie znaniy. - Saarbrucken, Germany: LAP LAMBERT Academic Publishing Gmbh&Co. KG, 2012.-196c.
[5] M. Ehrgott, Multicriteria Optimization, Springer, Heidelberg, 2005. doi:10.1007/3-540-27659-9
[6] A tutorial on multiobjective optimization: fundamentals and evolutionary methods Michael T. M. Emmerich1 • Andre´ H. Deutz1 Natural Computing https://doi.org/10.1007/s11047-018-9685-y
[7] Dynamic changes and multi-dimensional evolution of portfolio optimization Wei Zhou, Wenqiang Zhu, Yan Chen&Jin Chen Pages 1431-1456, Published online: 23 Aug 2021 Download citation https://doi.org/10.1080/1331677X.2021.1968308
[8] V. A. Perepelitsa, Mnogokriterialnye modeli i metody dlia zadach optimizatsii na grafah. — Saarbrücken : LAP LAMBERT Academic Publishing, 2013. – 337c.
[9] A. V. Bakurova, H. M. Ropalo, E. V. Tereschenko: Analysis of the Effectiveness of the Successive Concessions Method to Solve the Problem of Diversification. MoMLeT+DS 2021: 231-242 http://ceur-ws.org/Vol-2917/paper21.pdf - P. 231-242.
[10] Mathworks, 2019. MATLAB*-, 2019. URL: https://www.mathworks.com/campaigns/products.