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Study of Quadri Quintic Element Applicable to Deep Beams

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Abstract

A lot of studies have been performed to explore the use and application of conforming elements in meshing of two and three-dimensional structures. Here, a combination of 5 nodes on X axis and 3 nodes on Y axis has been considered. Hence, the element used here is a 12 noded rectangular nonconforming element, which can be termed as Quadri-Quintic nonconforming element. The development of a computer program to study the behavior of Quadri-Quintic nonconforming element and its application in meshing of deep beams has been targeted. The study may able to show that the proposed element gives results with higher accuracy and with faster convergence.

1 Introduction

Basically, the only method of structural analysis is the strain energy/ potential energy method. The applications of this method are developed as Slope deflection method, Moment distribution method, and Kani's method etc., basic concept of analysis being the same i.e. potential energy method. Finite element method is an extension of stiffness matrix method, which can be conveniently used for the analysis of two-dimensional and three-dimensional elements. The versatility of the finite element method has drawn much attention in the field of engineering. Numerous researches have been performed for the better and smarter applications of the method. As it involves lot of calculations, its growth is closely linked with the developments in computer technology. While solving a problem with FEM we come across repetitive work involving same formulae and calculations, as basic procedure remains same. In addition, working on large complex structure involves large matrices in Calculations. Keeping these points in mind a computer program has to be developed in the scope of the current work. Initially an excel spreadsheet is to be developed for simple meshing which can be extended for finer meshing using any programming language.

2 Objectives of the study

Basically, the concept of nonconforming element in the FEM is to be explored in this study. The behavior of Quadri-Quintic nonconforming element and its application in meshing of deep beams is to be studied. A computer program is to be developed using MS Excel for the same, which can be further extended for finer meshing using any programming language. A combination of Quadratic and Quintic i.e. 5 nodes on two sides and 3 nodes on other two sides are taken. This study may be able to obtain results with faster convergence.

3 Literature review

The plane quadrilateral element is a simple and widely used element in the analysis of plane problems and the design and development of low-order quadrilateral plane element with a high accuracy in coarse meshes has always been very popular in the Finite element method. Because of their flexibility and stability when compared with conforming finite element methods, nonconforming finite element methods have become very important and effective discretization methods for numerically solving, among others, high order elliptic problems, Stokes-like problems and Reissner-Mindlin plate bending problems. Compared with nonconforming triangular finite elements, it is more difficult to construct nonconforming quadrilateral finite elements^[3]. Huang and Xie^[2] analyzed a nonconforming 5 node transition element by finite element method in which the fifth node is placed at different positions as shown in figure 1.

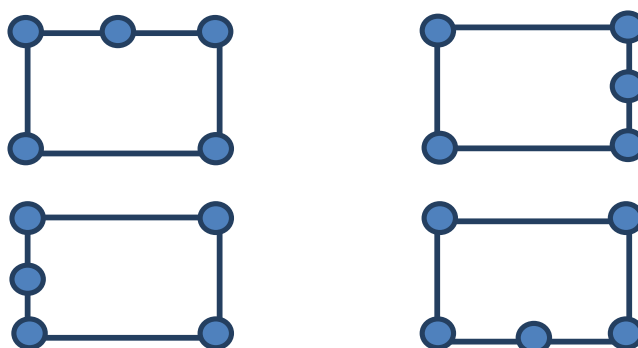


Figure 1: five noded transition element

Douglas N. Arnold and Gerard Awanou^[1] gave a new, simple, dimension-independent definition of the serendipity finite element family. **Hu Jun and Zhang Shang You**^[3] proposed two families of nonconforming finite elements of any odd order and one family of nonconforming finite elements of any even order on quadrilateral meshes. **Tinh Quoc Bui, Dam Quang Vo, Chuan zeng Zhang, Du Dinh Nguyen**^[7] developed a consecutive interpolation quadrilateral element CQ4 and studied its applications in the stress analysis of 2D elastic structures. **Juan Chen, Chong-Jun Li**^[5] presented a paper in which a 24-DOF quadrilateral spline element for couple stress/strain gradient elasticity is proposed, which is a single displacement function. **Tejpal Goda H., A. N. Desai, K. N. Sheth**^[6] carried out a study of nonconforming element with six-nodes and compared it with the conforming four noded rectangular element as shown in figure 2. **Jaimin A. Joshi, A. N. Desai, K. N. Sheth**^[4] carried out a study of nonconforming quadrilateral element with 10 nodes as shown in figure 3.

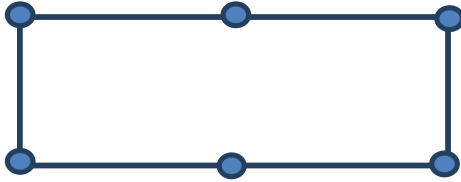


Figure 2: six noded rectangular element

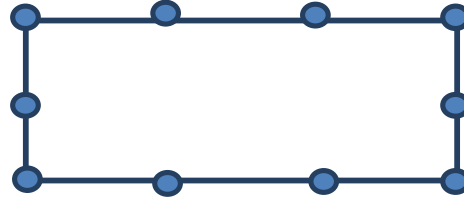


Figure 3: ten noded rectangular element

4 Quadri Quintic Nonconforming Element

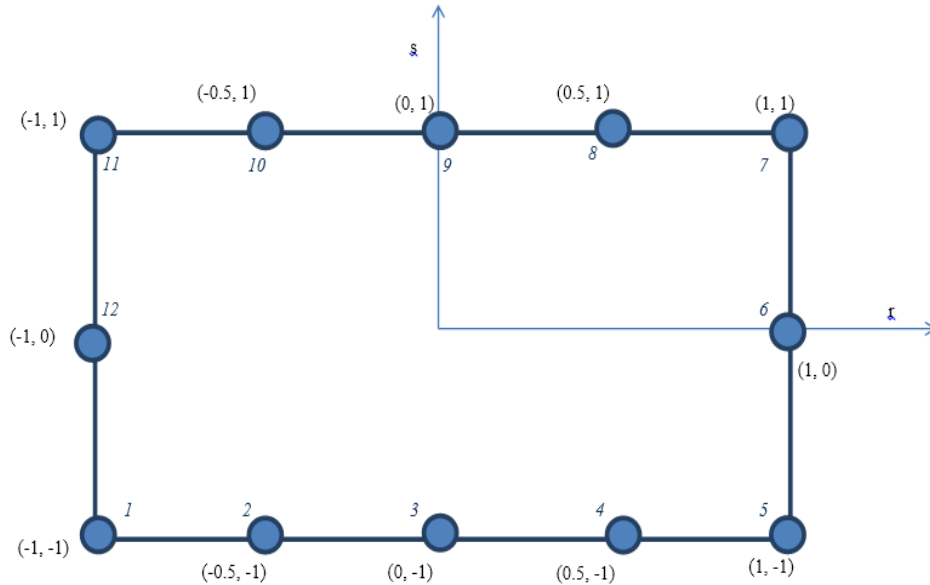


Figure 4: Quadri Quintic nonconforming element

Nonconforming elements are those, which have asymmetric distribution of nodes along both the axes. For example, adding three more nodes at equal intervals on the horizontal sides and one middle node on vertical sides of a basic four-noded quadrilateral will make it a 12 noded nonconforming rectangular element. Hence, adding and numbering 8 more nodes in anti-clockwise direction to make it a Quadri-Quintic nonconforming element as shown in figure 4, considering any one node, the value of its shape function at that particular node should be one and zeros at all other nodes. In addition, summation of all the shape functions should come to unity. Shape functions for this 12-noded Quadri Quintic element can be derived as-

$$N_2 = \frac{-4r(1-s)(1-r^2)(\frac{1}{2}-r)}{3}$$

$$N_3 = 2(1-r^2)(1-s)(\frac{1}{4}-r^2)$$

$$N_4 = \frac{4r(1-s)(1-r^2)(\frac{1}{2}+r)}{3}$$

$$N_6 = \frac{(1+r)(1-s^2)}{2}$$

$$N_8 = \frac{4r(1+s)(1-r^2)(\frac{1}{2}+r)}{3} \qquad N_{10} = \frac{-4r(1+s)(1-r^2)(\frac{1}{2}-r)}{3}$$

$$N_9 = 2(1-r^2)(1+s)(\frac{1}{4}-r^2) \qquad N_{12} = \frac{(1-r)(1-s^2)}{2}$$

If N'_1, N'_2, N'_3 and N'_4 are shape functions for corner nodes of a standard four noded quadrilateral element, shape functions for the corner nodes of a Quadri Quintic element can be revised as,

$$N_1 = N'_1 - \frac{N_{12}}{2} - \frac{3N_2}{4} - \frac{N_3}{2} - \frac{N_4}{4} \qquad N_5 = N'_5 - \frac{N_6}{2} - \frac{3N_4}{4} - \frac{N_3}{2} - \frac{N_2}{4}$$

$$N_7 = N'_7 - \frac{N_6}{2} - \frac{3N_8}{4} - \frac{N_9}{2} - \frac{N_{10}}{4} \qquad N_{11} = N'_{11} - \frac{N_{12}}{2} - \frac{3N_{10}}{4} - \frac{N_9}{2} - \frac{N_8}{4}$$

Now, suitability of this element to application to the deep beam analysis can be understood by considering a few points. Deep beams are subjected to non-linear stress distribution even in the elastic stage. It can be observed from the shape functions of the Quadri Quintic element that it can serve for the non-linear distribution. While deep beam behaves as a two dimensional member, the proposed nonconforming element is also a two dimensional element. First step in analysis is to find field variables. It is assumed that at any point inside the element, field variable is a function of field variables at nodal points of the element. The function that relates the field variables of internal points to the field variables at nodal points is called shape function or interpolating function. For two-dimensional stress analysis the basic field variable is displacement.

$$u = \sum N_i u_i \qquad v = \sum N_i v_i$$

Here ‘i’ is the index number for nodes and N_i is the shape function for i^{th} node. u and v denote the horizontal and vertical displacements respectively.

$$u = \sum N_i u_i = N_1 u_1 + N_2 u_2 + N_3 u_3 + \dots + N_{12} u_{12}$$

$$v = \sum N_i v_i = N_1 v_1 + N_2 v_2 + N_3 v_3 + \dots + N_{12} v_{12}$$

$$\text{i.e. } \{u\} = [N] * \{\delta\}$$

Where,

$$[N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & \dots & N_{12} & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & \dots & 0 & N_{12} \end{bmatrix}$$

And displacement,

$$\{u\} = [N] * \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ \cdot \\ \cdot \\ u_{12} \\ v_{12} \end{Bmatrix}$$

Now, the Jacobian matrix can be written as

$$[J] = \begin{bmatrix} \partial N_1 / \partial r & \partial N_2 / \partial r & \dots & \partial N_{12} / \partial r \\ \partial N_1 / \partial s & \partial N_2 / \partial s & \dots & \partial N_{12} / \partial s \end{bmatrix} * \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ \cdot & \cdot \\ \cdot & \cdot \\ x_{12} & y_{12} \end{bmatrix}$$

Here, (x_i, y_i) are the geometric coordinates of the nodes. r and s are the isoparametric axes.

The Geometric matrix will be

$$[G] = \begin{bmatrix} \partial N_1 / \partial r & 0 & \dots & \partial N_{12} / \partial r & 0 \\ \partial N_1 / \partial s & 0 & \dots & \partial N_{12} / \partial s & 0 \\ 0 & \partial N_1 / \partial r & \dots & 0 & \partial N_{12} / \partial r \\ 0 & \partial N_1 / \partial s & \dots & 0 & \partial N_{12} / \partial s \end{bmatrix}$$

Strain-Displacement matrix is given by $[B] = [A][G]$

Where

$$[A] = \begin{bmatrix} J_{11}^* & J_{12}^* & 0 & 0 \\ 0 & 0 & J_{21}^* & J_{22}^* \\ J_{21}^* & J_{22}^* & J_{11}^* & J_{12}^* \end{bmatrix}$$

The stiffness matrix is obtained by the following equation,

$$[K] = t * \int_{-1}^1 B^T D B dA$$

Where, t = width of the beam

D = 3*3 constitutive matrix

dA = $dx dy$

= $|J| * dr * ds$

In this study, the *Gaussian quadrature approach of integration (three points)* is used. This method has proved most useful in finite element work. The form of evaluating I is shown below. Extension to two and three dimensions follows readily. Considering n point approximation,

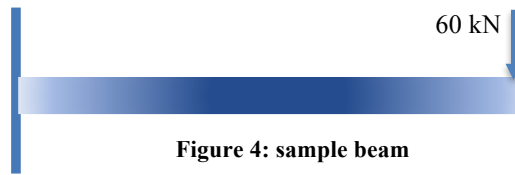
$$I = \int_{-1}^1 f(\varphi) d\varphi = w_1 f(\varphi_1) + w_2 f(\varphi_2) + \dots + w_n f(\varphi_n)$$

Where $w_1, w_2 \dots w_n$ are the weights and $\varphi_1, \varphi_2, \dots, \varphi_n$ are the sampling points or Gauss points. Gaussian Quadrature of n points will provide an exact answer if f is the polynomial of order $(2n-1)$. The Gauss points are located symmetrically with respect to origin and symmetrically placed points have the same weights. For a Quadri Quintic element, size of the geometric matrix G is 4×24 and the size of stiffness matrix is 24×24 .

5 Sample Meshing

First sample meshing is done for a cantilever deep beam using a single element. The beam is 1 m long, 0.5 m deep and 0.15 m wide. The beam is subjected to a point load at the free end. The procedure of finite element analysis is performed by the program developed in MS Excel. Three point Gauss Quadrature approach is used for the integration. Values of deflections are obtained at each node and compared with the standard deflection value for the cantilever under point load. i.e.

$$\delta = \frac{PL^3}{3EI}$$

**Figure 4: sample beam**

	MS-EXCEL	FORMULA
DEFLECTION	0.525 mm	0.467 mm

Table 1: results

The percentage error occurred here is 12.3%. The difference in the results by both methods can be explained by the fact that the standard deflection formula does not count for shear deformations whereas present study has considered the shear deformations.

6 Conclusion

It is concluded that the proposed Quadri Quintic Element is stable and applicable to analysis of deep beams. Higher degree of accuracy may be obtained with finer meshing.

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